Estimation of the constant π using a statistical method

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A statistical method was used to estimate the value of the constant π. An accuracy better than ± 0.0002 was achieved from 10×10^7 samples.

PACS numbers:

THE METHOD

Random numbers for the x and the y coordinates of a point will be chosen randomly between zero and one:

$$0 < x < 1 \qquad 0 < y < 1 \,.$$

These numbers will be scattered in a total area of 1. (See Figure 1.) The area of the circle inside this quadrant is $\pi/4$. The points will then fall into the circular region with a probability of $\pi/4$. By counting the points that do fall into the circle and finding their ratio to total number of points gives a statistical estimate for $\pi/4$.

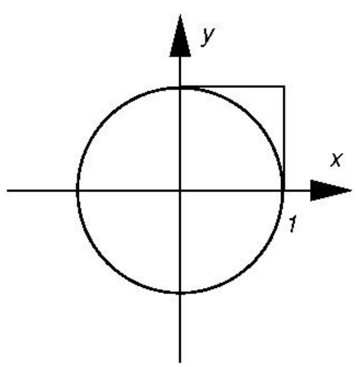

FIG. 1: The geometry of the setup. Random points are generated in the indicated square within $0 < x, y < 1$.

THE COMPUTATIONAL RESULTS

N random points were selected, and the those coocrdinates which satisfy the condition $x^2 + y^2 < 1$ were counted. The estimate for the number π is then

$$p = 4n/N. \tag{1}$$

This procedure was repeated 10 times, thus resulting in 10 values of p_i according to Equation 1. The average $\overline{p}$ of these 10 values is the estimate for π:

$$\overline{p} = \frac{1}{10}\sum_{i=1}^{10} p_i \qquad\qquad \overline{p^2} = \frac{1}{10}\sum_{i=1}^{10} p_i^2$$

N	$\overline{p}$	$\sigma/\sqrt{9}$
10^2	$3.1\overline{2}$	0.045
10^3	$3.1\overline{38}$	0.011
10^4	$3.1\overline{468}$	0.0058
10^5	$3.14\overline{1}$	0.0013
10^6	$3.141\overline{8}$	0.00062
10^7	$3.141\overline{7}$	0.00015

TABLE I: The values for number of points N, the estimate $\overline{p}$, and the reliability of the result, $\sigma/\sqrt{9}$. (The bar over the digits of $\overline{p}$ indicate uncertain values.)

The standard deviation σ of these 10 values were then used as a measure of the possible error in the result:

$$\sigma^2 = \frac{1}{9}\left(\overline{p^2} - \overline{p}^2\right).$$

Table I shows the estimate for π and possible error versus number N. A result of $\pi \sim 3.141\overline{7}$ with an accuracy (70% confidence) of ± 0.00015 was obtained when using $10N = 10^8$ points.

APPENDIX - THE COMPUTER CODE

The computer code was implemented in *Python*:

```
import numpy as np
from random import *

# Number of samples will be 100, 1000, ...
n=10;
for nn in range(1,5):
  n=10*n;

  sum=0;
  ssum=0;

# The procedure will be repeated 10 times
  for ii in range(1,10):
    count=0;

    for i in range(1,n):
# generate the coordinates
      x=random();
      y=random();

# check if inside the circle:
      if x*x + y*y < 1 :
        count=count+1;

# estimate for pi:
    p = 4*count/n;
# find average p:
    sum = sum  + p;
# find average of p*p
    ssum= ssum + p*p;

  aver = sum/10;
  sigma = np.sqrt(( ssum/10 -aver*aver )/9);
# write the result
  print ("For n , aver, sigma")
  print ( n, aver , sigma )
```