Quantum Entanglement and Bell Inequalities: CHSH Inequality as a special case

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Abstract- In this paper the phenomenon of quantum entanglement is studied from its first appearance in the EPR paper(1935). Its historical progress and development is mentioned briefly and the non-locality argument is made by the use of a simplified version of CHSH inequality. In addition, comparison with classical models is made by using negative probability distribution theory. As conclusion, the need for a quantum mechanical model is discussed with respect to Einstein's description of reality and locality argument.

The introduction of the idea of entanglement in the EPR paper of 1935[1] caused the concept of locality to become a main point of argument for and against the quantum theory. Einstein used this idea to argue that quantum theory is not complete and there is a need for a new theory, which were later named and recognized as "hidden variable theories" [2].

In 1964 John Bell[3] published his paper on how the entire family of local hidden-variable theories are inconsistent with the predictions of quantum mechanics. By "Bell's theorem" he shows that for that type of a system to operate it would need instantaneous communication between the measuring devices which violates the original assumptions of local realism.

Later Clauser, Horne, Shimony and Holt (CHSH paper of 1969)[4] generalized Bell's ideas to apply to realizable experiments.

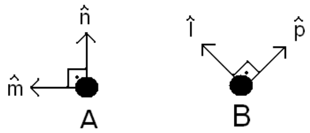
In this paper a simplified version of the CHSH argument is studied through using quantum mechanical formalism. A one qubit system is taken into account,

$$|\Psi>_{AB} = 1/\sqrt{2}\ [\ |\uparrow\downarrow>_{AB} - |\downarrow\uparrow>_{AB}]$$

With,

$$|\uparrow\downarrow>_{AB} = |\uparrow>_A \otimes |\downarrow>_B$$
$$|\downarrow\uparrow>_{AB} = |\downarrow>_A \otimes |\uparrow>_B$$

A system of the following type is considered with the measurement directions m, n, l and p.



Later on it is shown that for a realistic theory a measurement such as:

$$S = <(S_{An}+S_{Am})S_{Bl} + (S_{An}-S_{Am})S_{Bp}>$$

Will have expectation values in the range of $-2 \leq S \leq 2$ while the quantum mechanical prediction values can violate this inequality.

In the following section, the basic assumptions which led to the construction of the classical system is taken into consideration. A probability distribution function such as,

$$p(S_{An}, S_{Am}, S_{Bl}, S_{Bp})$$

Is defined where the S values represent the expectation values of measurement taken in the related direction. The assumptions:

$$p(S_{An}, S_{Am}, S_{Bl}, S_{Bp}) \geq 0$$

$$\sum_i p(s_i) = 1$$

Are made with accordance to the basic assumptions of realistic theories.

Later on the assumption of $p(S_{An}, S_{Am}, S_{Bl}, S_{Bp}) \geq 0$ will be shown to be violated if the predictions of quantum mechanics are taken into consideration. Therefore the predictions of quantum mechanics are inconsistent with not just local realistic theories but realistic theories in general.

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