UNDERSTANDING THE FOURIER TRANSFORMS OF LENSES FOR IMAGING PURPOSES

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Introduction

In the first quarter of the 19th century, French physicist Augustin Jean Fresnel studied and made the classical theory of diffraction come out in his study. William Rowan Hamilton, Gustav Kirchhoff, George Biddell Airy, John William Strutt (Lord Rayleigh), Ernst Abbe and Arnold Sommerfeld who were outstanding scientists and made a success of the wave nature of light had accepted and used Fresnel's concepts. In addition, Josef von Fraunhofer generated diffraction gratings that helped to the understanding of light diffraction and Jean Baptiste Joseph Fourier identified that periodic functions can be regarded as sums of sinusoids and also accepted harmonic analysis as the basis of Fourier optics **[1]**.
Fourier optics features the meaning of the propagation of light wave on the basis of harmonics analysis and linear systems. Harmonic analysis is the Fourier transform whose methods can be used to analyse signals and systems in several areas **[2]**. Linear systems are used for formulating diffraction as well as imaging. If there is an input, the system forms an output. Therefore, it can be understood that the system is an input-output mapping **[3]**.

Fourier Transforming With A Lens

A converging lens can achieve two-dimensional Fourier transformations. In any optical system we can use converging lens as a Fourier tranforming material.

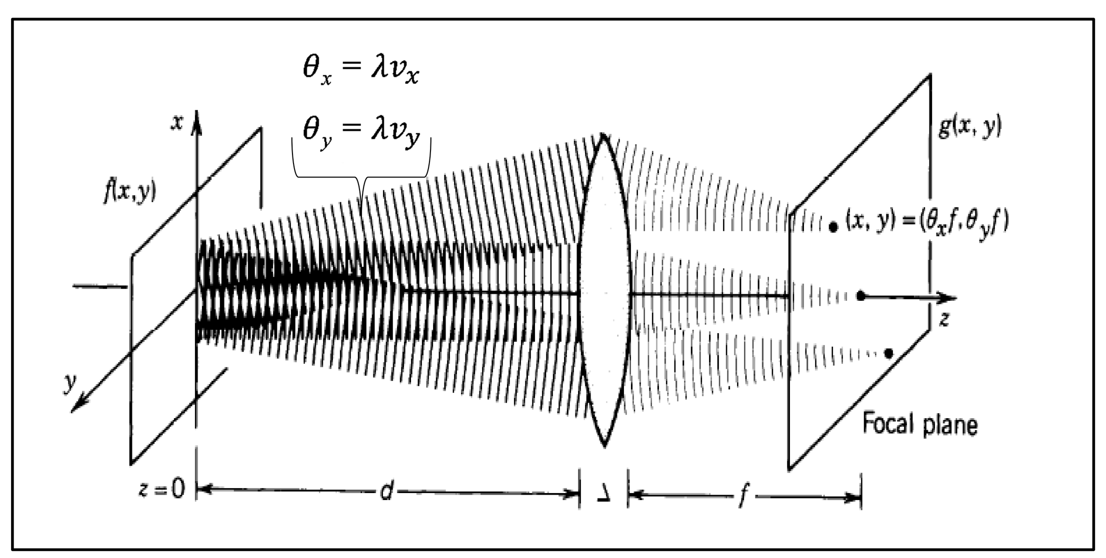

Fig. 1. Fourier transform using a lens **[2]**

$$g(x,y) = h_l e^{\left(\frac{j\pi(x^2+y^2)(d-f)}{\lambda f^2}\right)} F\left(\frac{x}{\lambda f}, \frac{y}{\lambda f}\right)$$

Where $h_l = \frac{j}{\lambda f} e^{[-jk(d+f)]}$ and $\mathbf{F}(\frac{\mathbf{x}}{\lambda \mathbf{d}}, \frac{\mathbf{y}}{\lambda \mathbf{d}})$ is the Fourier transform of the input plane $\mathbf{f}(\mathbf{x}, \mathbf{y})$.

If we think absolute value of the focal plane $\mathbf{g}(\mathbf{x}, \mathbf{y})$, it is obvious that the intensity does not depend on the distance **d**.

$$I(x,y) = \frac{1}{\lambda f^2} \left|F\left(\frac{x}{\lambda f}, \frac{y}{\lambda f}\right)\right|^2$$

Moreover, if we put the the object on the focal length of the lens **(d = f)**, the phase curvature disappears meaning that the exact Fourier transform relation.

$$g(x,y) = h_l F\left(\frac{x}{\lambda f}, \frac{y}{\lambda f}\right)$$

Experiment

The 4F Optical System consists of the input plane where we put the object, first Fourier transform lens, the Fourier plane, second Fourier transform lens and the output plane where we get the image. We make the distance between each of them F(25cm) that is the focal length of the Fourier transform lenses.
We use a monochromatic light as a light source which is Helium-Neon Laser and thus at the input plane we have to use the transparent objects instead of a photograph. We record the profiles of both the Fourier and output planes of these objects by using the CCD Camera.

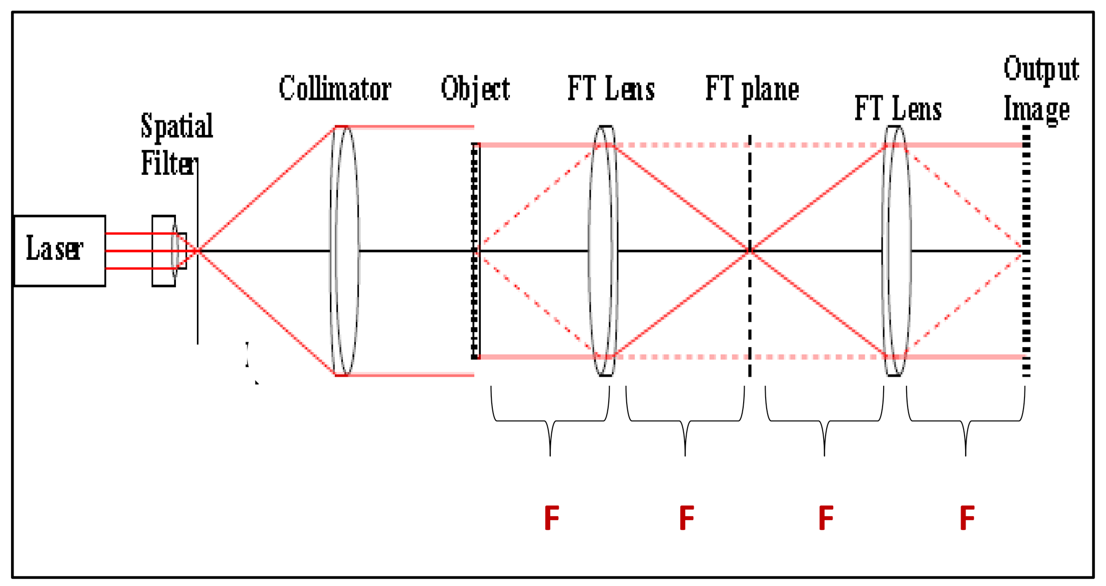

Fig. 2. 4F Optical System **[4]**

Measurements

We use CCD Camera to get the profiles of the Fourier and output planes of the objects. In addition, we derive benefit from the C Sharp Programming Language for the same purpose that is used for the comparison of the profiles.

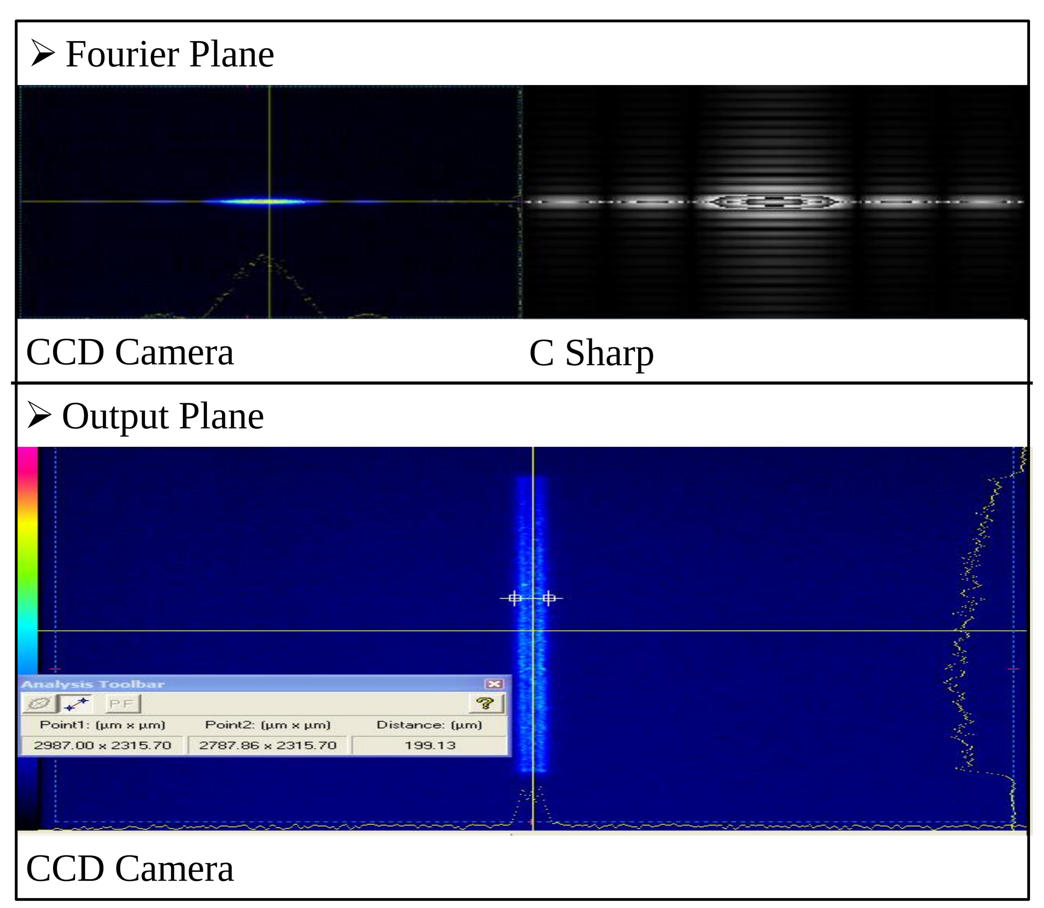

Fig. 3. 200μm Rectangular Slit

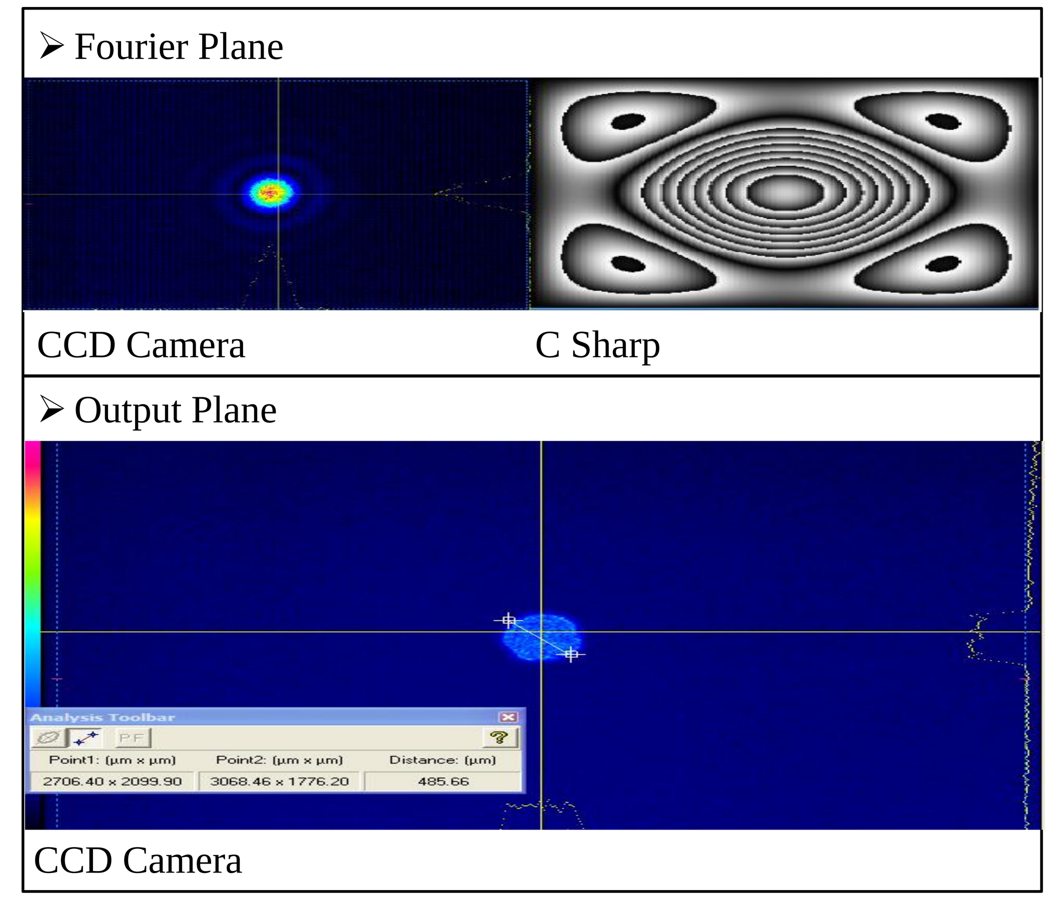

Fig. 4. 500μm Circular Slit

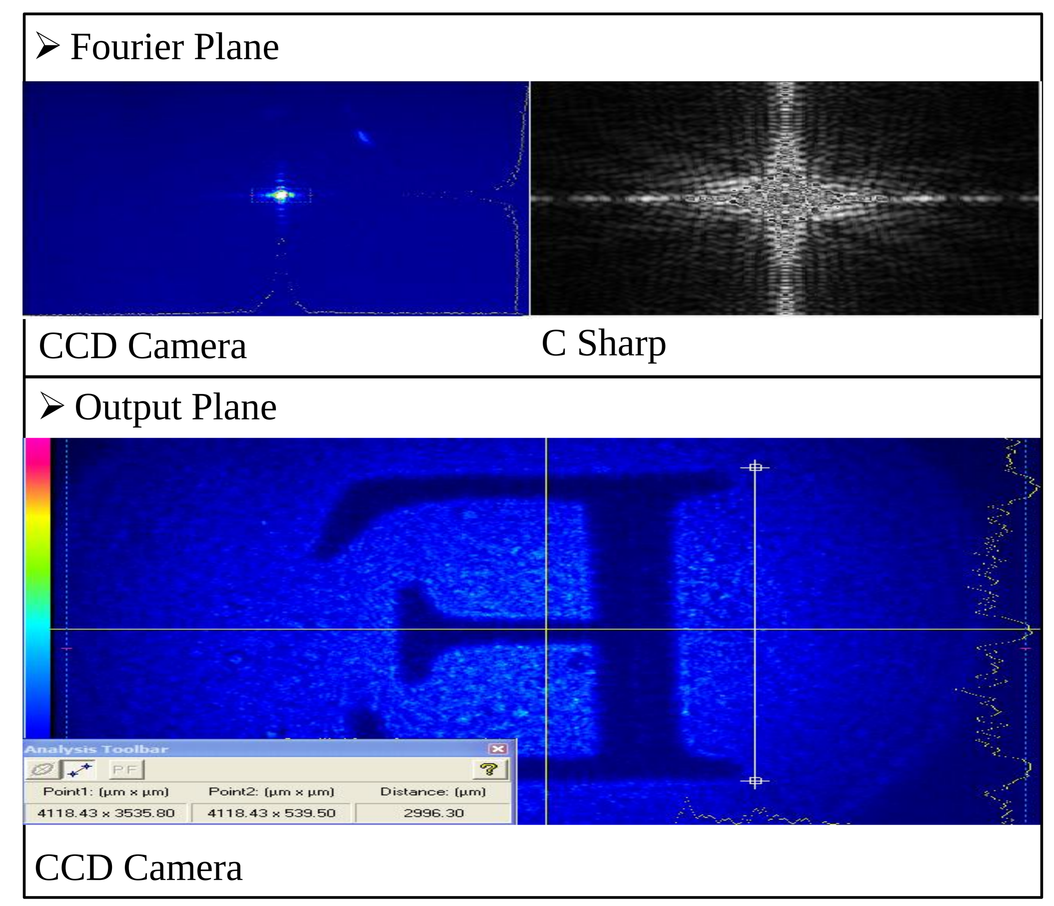

Fig. 5. Transparent Object "E"

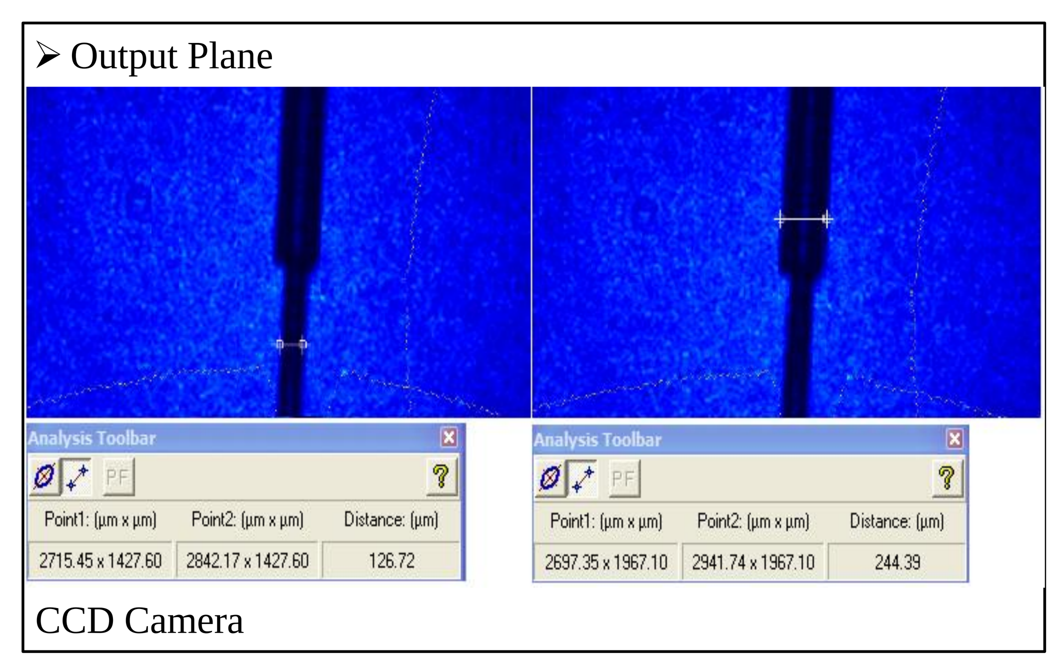

Fig. 5. Multi-Mode Optical Fiber

Conclusions

In this paper we have shown both theoretical and experimental processes of Fourier transforms of lenses for imaging purposes. Firstly, we explained how a lens can be used as a phase transformation and a Fourier transform tool. We see that when we apply a Fourier transform to any object with the help of a lens, we can get only amplitude information correctly. On the other hand, it is clear that we can also get phase information; however, it is not suitable and understandable. Therefore, as a second step, we have construct the 4F optical system with spatial filtering and collimating processes to use two identical lenses in order to get clear phase and amplitude information. In this step, we choose transparency for the input plane objects which are rectangular and circular slits, transparent object and optical fibers because our laser has monochromatic light source and hence we can not use an object like a photograph. If we want to explain shortly what the 4F optical system is, we can say that it has a Fourier plane at the middle of the setup and this Fourier plane gives us the focused diffraction pattern of the input plane objects. When we look at the Fourier plane with our eyes, we can not see something meaningful; however, when we used CCD Camera or C# programme, we can see the focused diffraction pattern clearly. Lastly, at the output plane, we gained the replica images of the input plane objects as we proof before theoretically. Our system works so well that at the output plane we can measure width or length of the input plane objects with very little %error. In the ligth of this result, it is possible that we can measure the width or length of anything that is proportional to the micrometer size such as fingerprint, hair, blood cells. In this experiment, we have satisfied that without any complex computer or digital systems we can do several important image processing with the help of the 4F optical system. In addition, we have given examples by using C# programme language for the purposes of image proccessing instead of Matlab and this FFT programme can be developed and it can be used much more in the future..

Literature cited

[1]Mansuripur, M. (2000). Fourier optics, part 1. *Optics and Photonics News*, *11* (5), 53.
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