2D Bloch Wave Optics

or the

Peculiar Properties of Light in periodic media

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http://www.ioip.mpg.de/~russell/PC_website/index.htm

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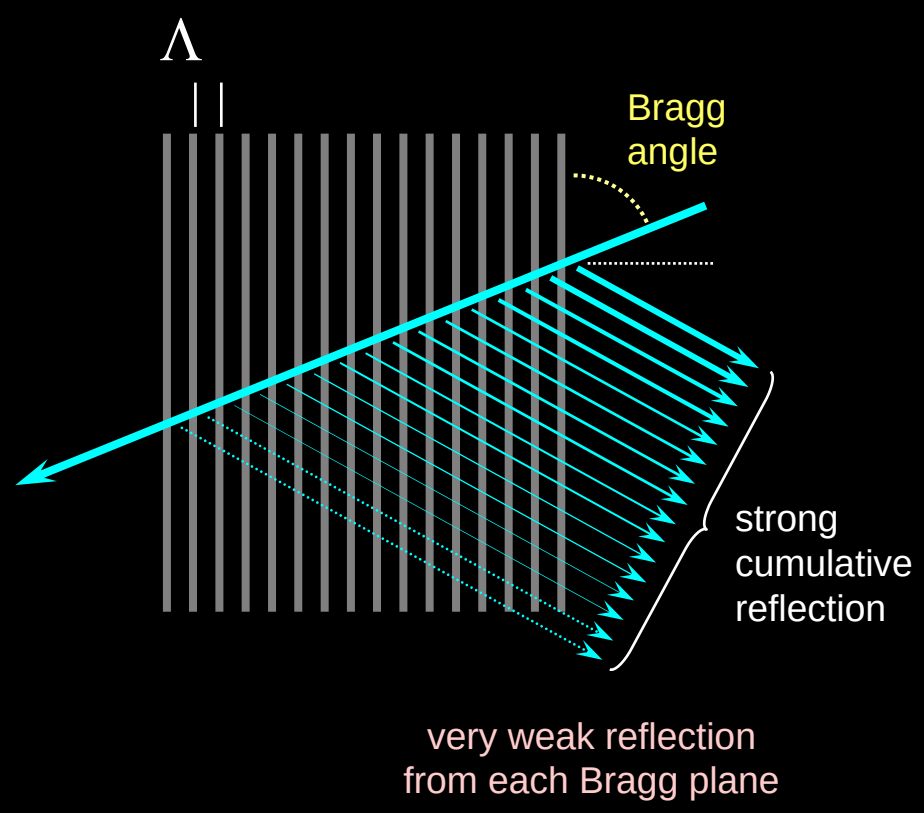
Topics

- Peculiar Bloch waves
  - nearly free photon model
  - wavevector diagrams
  - anatomy of a Bloch wave
  - negative & positive refraction
  - interference, Green's functions
  - curious focusing device
  - TIR at normal incidence
  - slowness diagrams & diffraction
  - square lattices
- Conclusions

“Nearly free photon” theory



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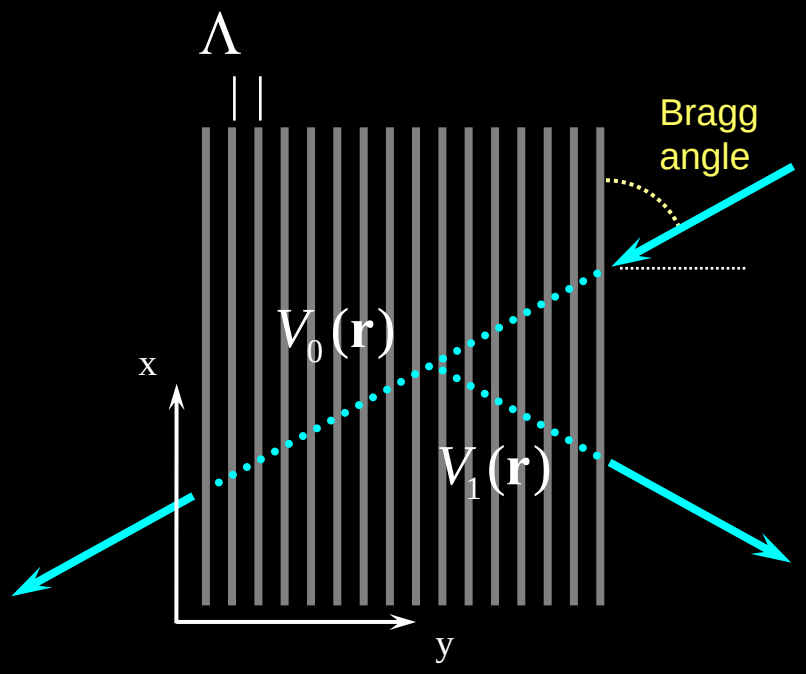
"Nearly free photon" theory

[a.k.a. coupled mode theory]



- assume that waves exist inside the periodic region
- allow them to be "coupled" by Bragg scattering:

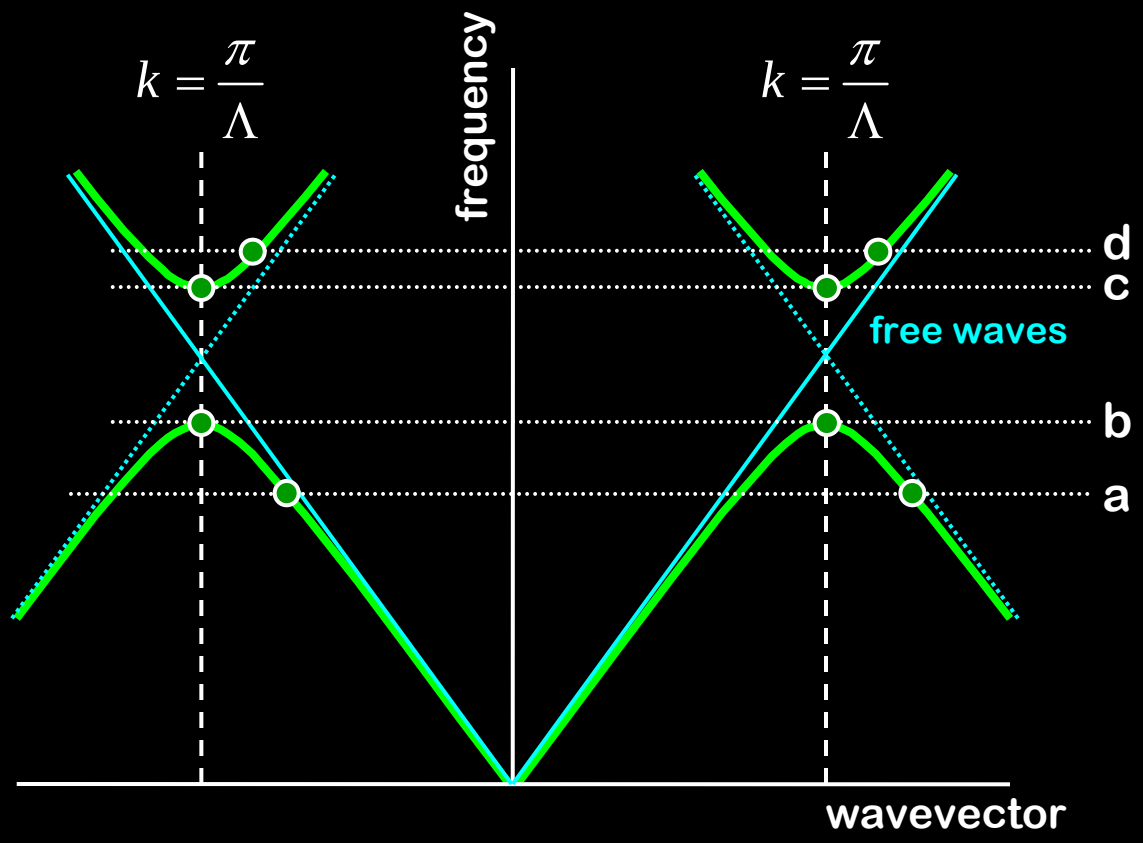
Bloch wavevector

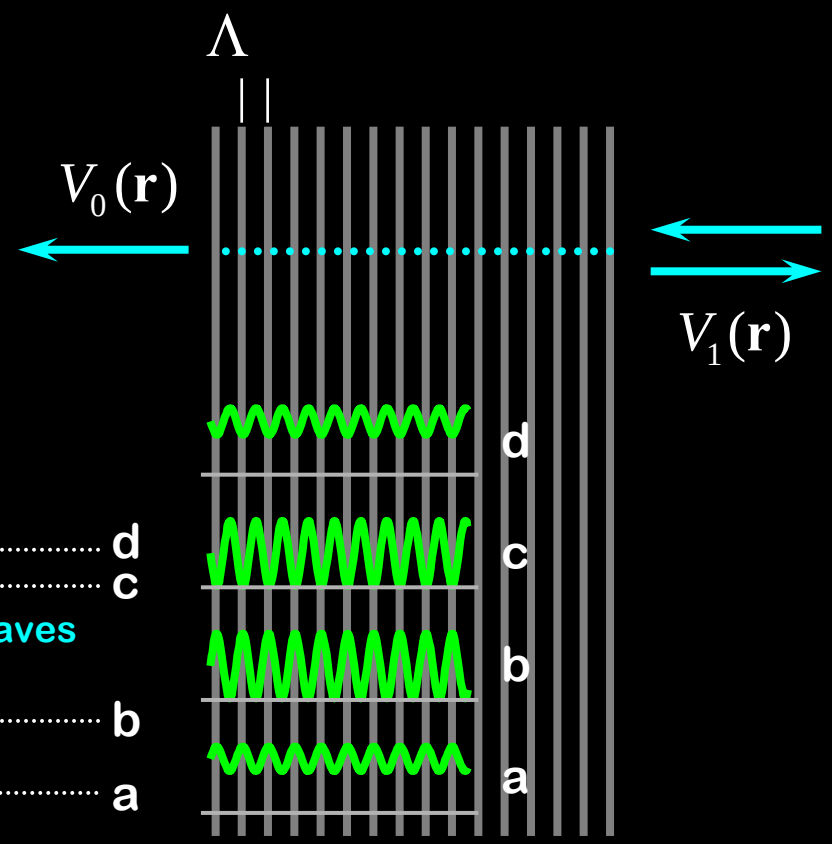
$$E(\mathbf{r}) = V_0(\mathbf{r})\exp[i\,\mathbf{k}_\mathrm{B}\cdot\mathbf{r}] + V_1(\mathbf{r})\exp[i\,(\mathbf{k}_\mathrm{B}-\mathbf{K})\cdot\mathbf{r}]$$

$$\frac{2\pi}{\Lambda}\hat{\mathbf{y}}$$

- at a particular frequency, special values of Bloch wavevector yield:
  - a "magic" combination of amplitudes that "sneaks through" the structure without change
  - these are the Bloch waves

Solutions for a 1-D crystal





reminder: $\mathbf{v}_G = \frac{\partial \omega}{\partial \mathbf{k}}$

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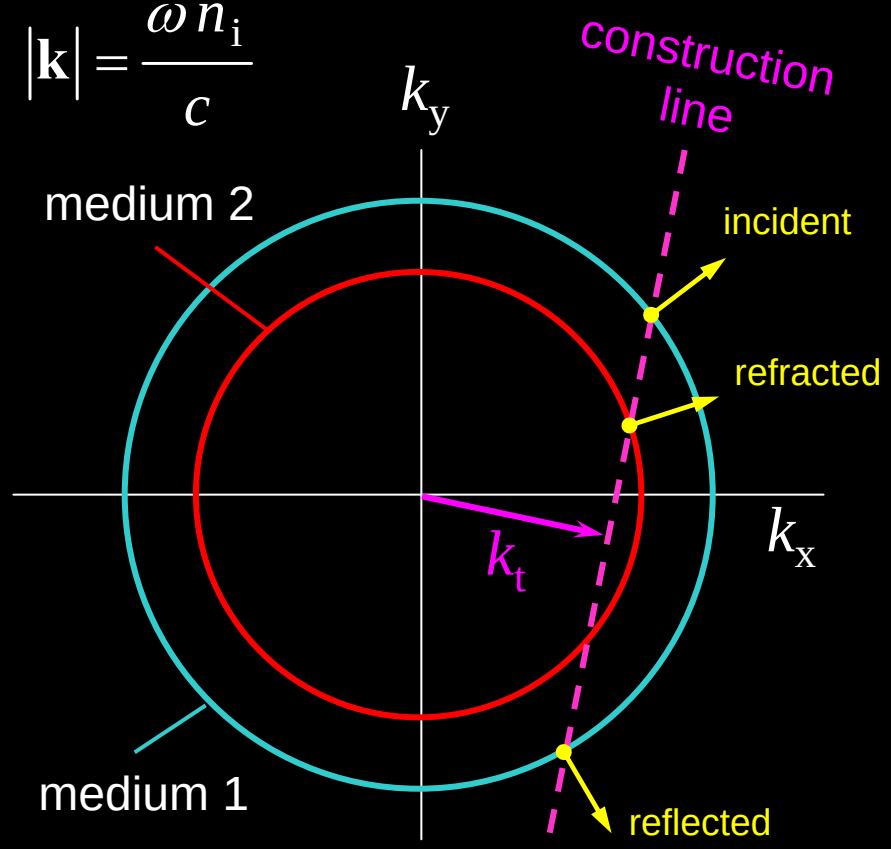
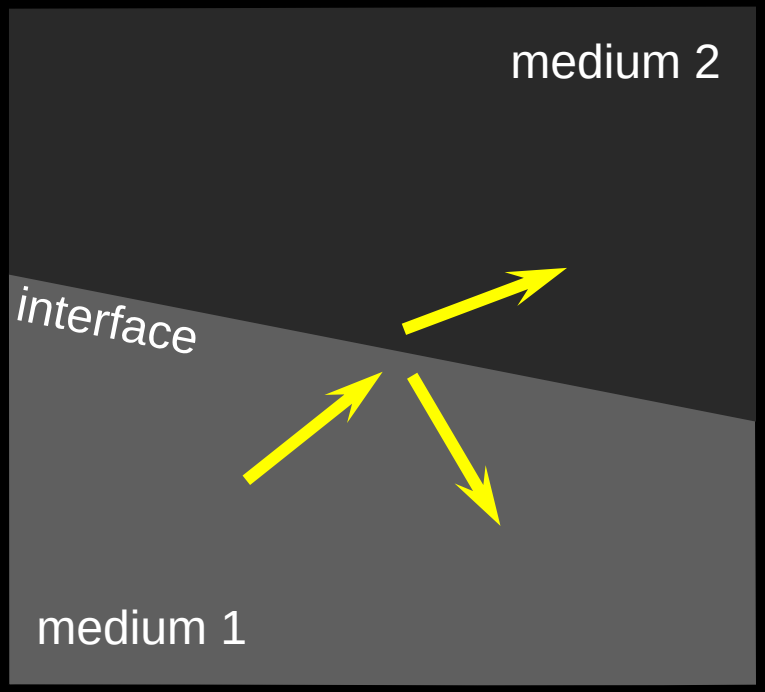
isotropic media

Refraction & reflection

$$|\mathbf{k}| = \frac{\omega n_{\mathrm{i}}}{c}$$



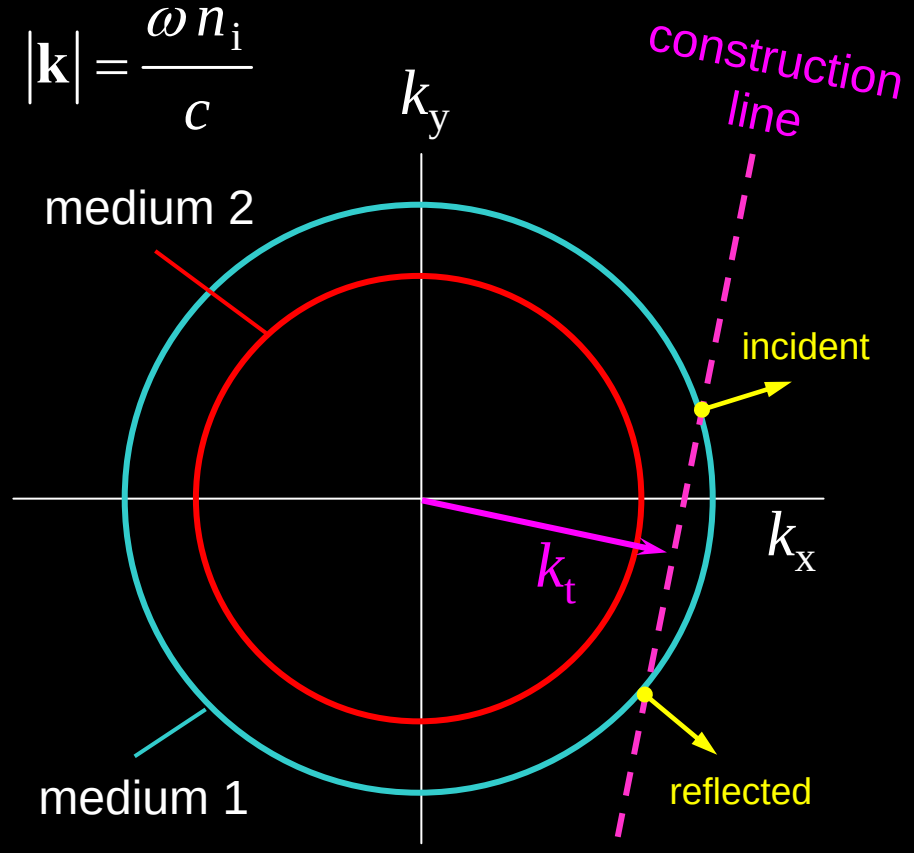
wavevector diagram



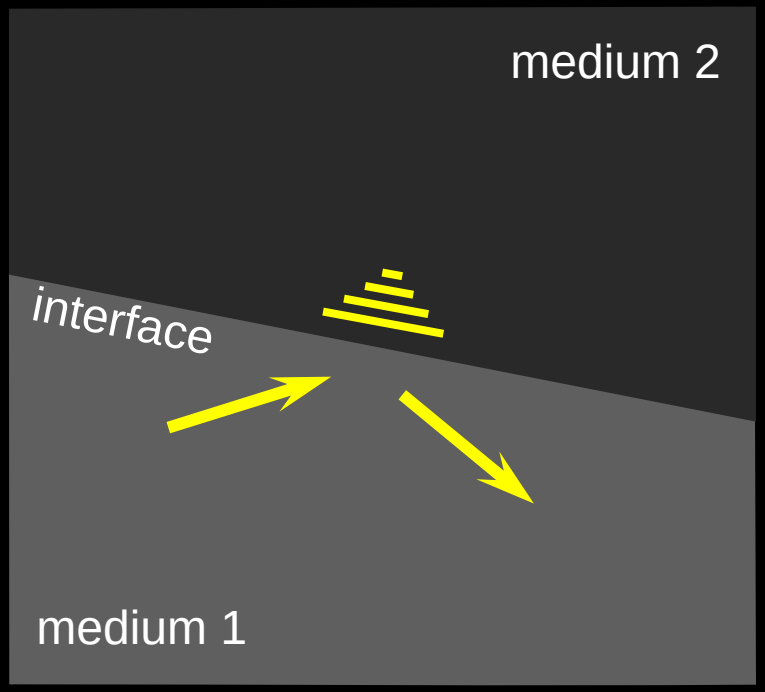
ray diagram

Total internal reflection

$$|\mathbf{k}| = \frac{\omega n_{\mathrm{i}}}{c}$$



wavevector diagram



ray diagram

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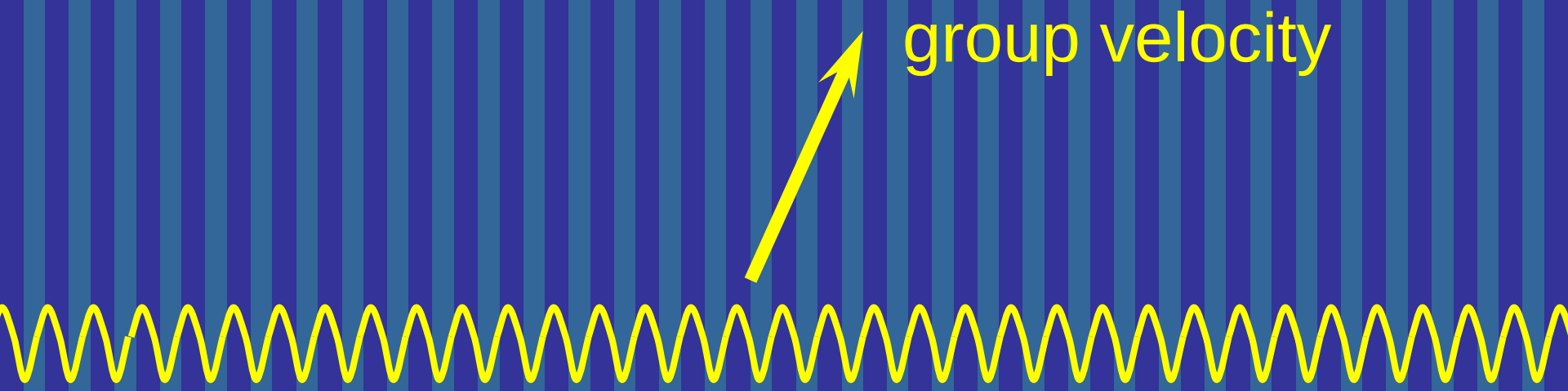
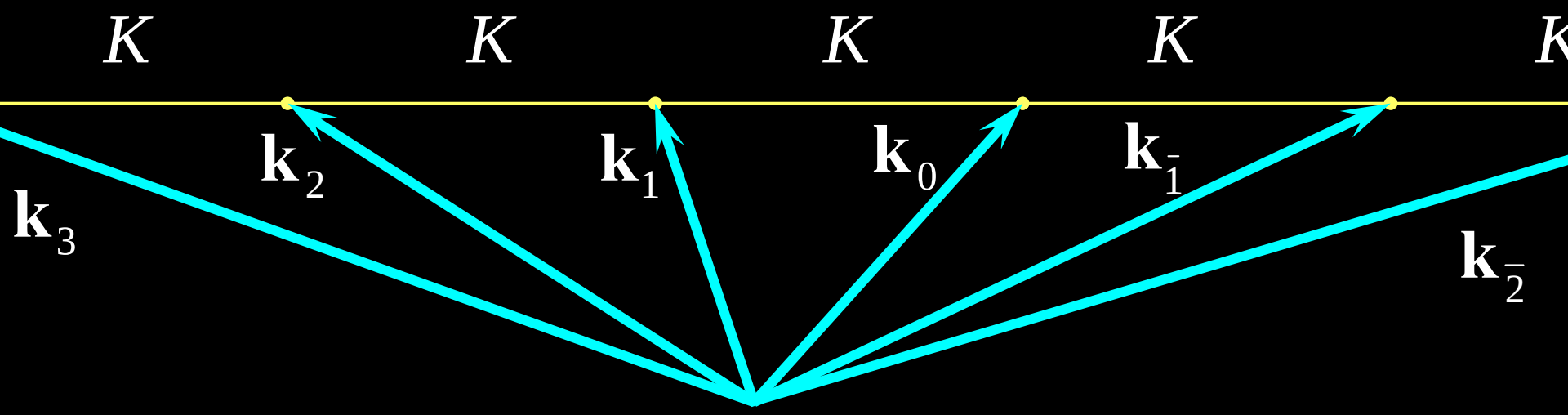
anatomy of a Bloch wave: Real space



$$K = 2\pi / \Lambda$$

Λ

anatomy of a Bloch wave: Reciprocal space



$$K = 2\pi / \Lambda$$

Bloch's theorem

reciprocal lattice vector

$$\Psi(\mathbf{r},\omega) = \exp(-j\,\mathbf{k}_\mathrm{B}\cdot\mathbf{r})\sum_m A_m \exp(-jm\mathbf{K}\cdot\mathbf{r})$$

$$= \exp(-j\,\mathbf{k}_\mathrm{B}\cdot\mathbf{r})\;P(\mathbf{r})$$

$$|\mathbf{K}| = 2\pi/\Lambda$$

lattice spacing

Bloch wavevector

function with period of lattice

Bloch waves have *multiple* phase velocities & a *single* group velocity

$$v_{\phi m} = \omega/(k_B + mK)$$

$$\mathbf{v}_g = \nabla_\mathbf{k}\,\omega(\mathbf{k}_B)$$

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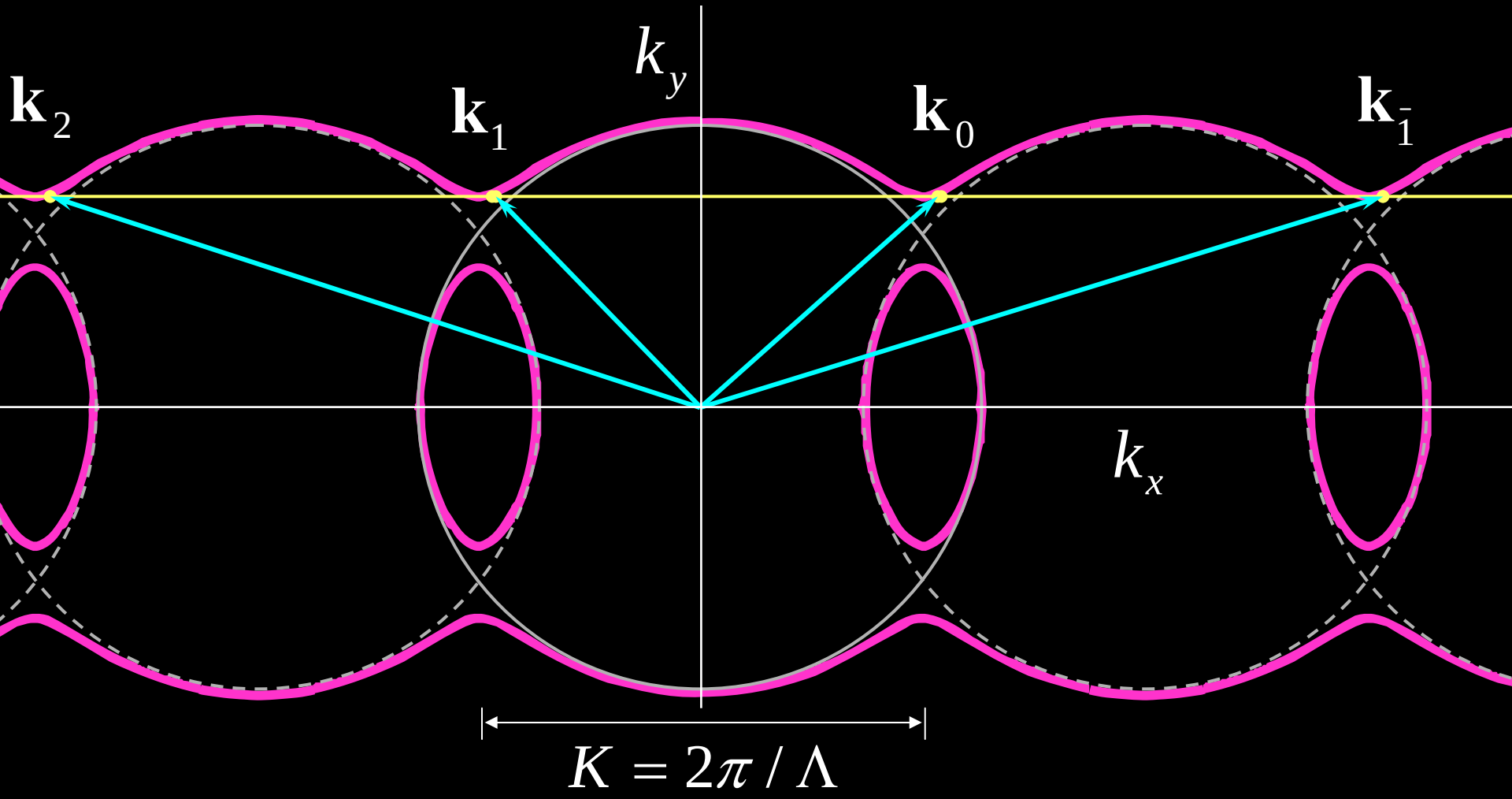
Bone-structure & face

with flesh

without flesh

http://tutorialblog.org/skull-face/

anatomy of a Bloch wave



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Hugely enhanced design freedom ...

magnitude & direction of group & phase velocity can be almost independently controlled

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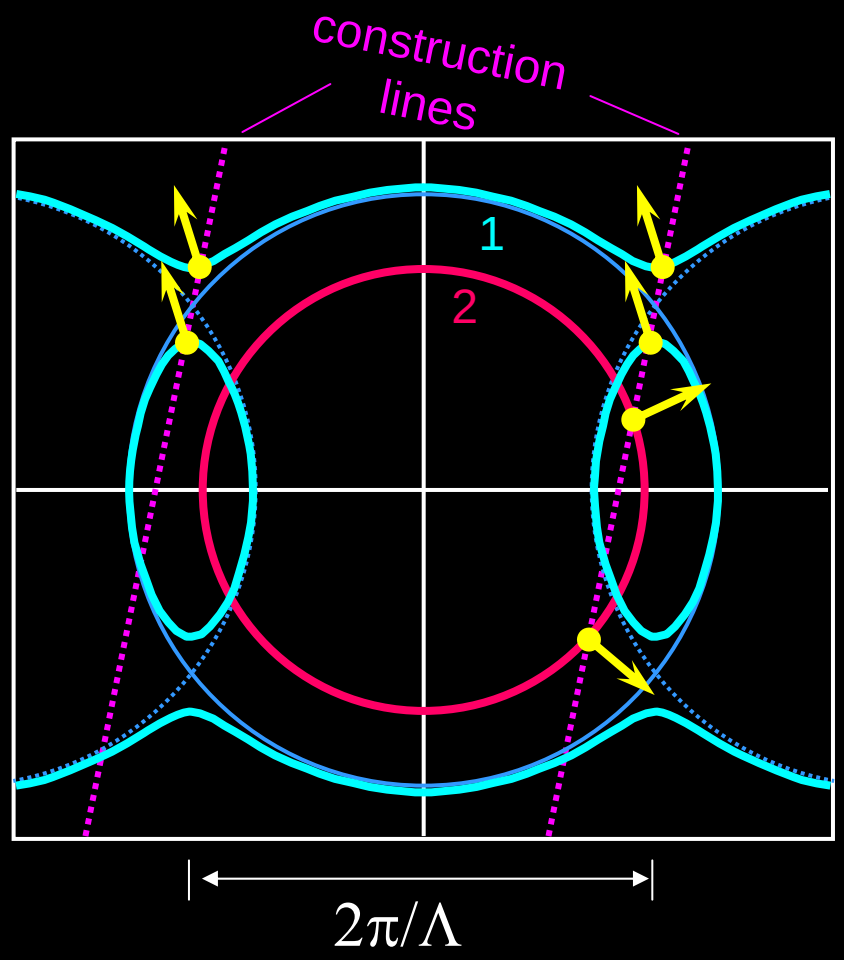
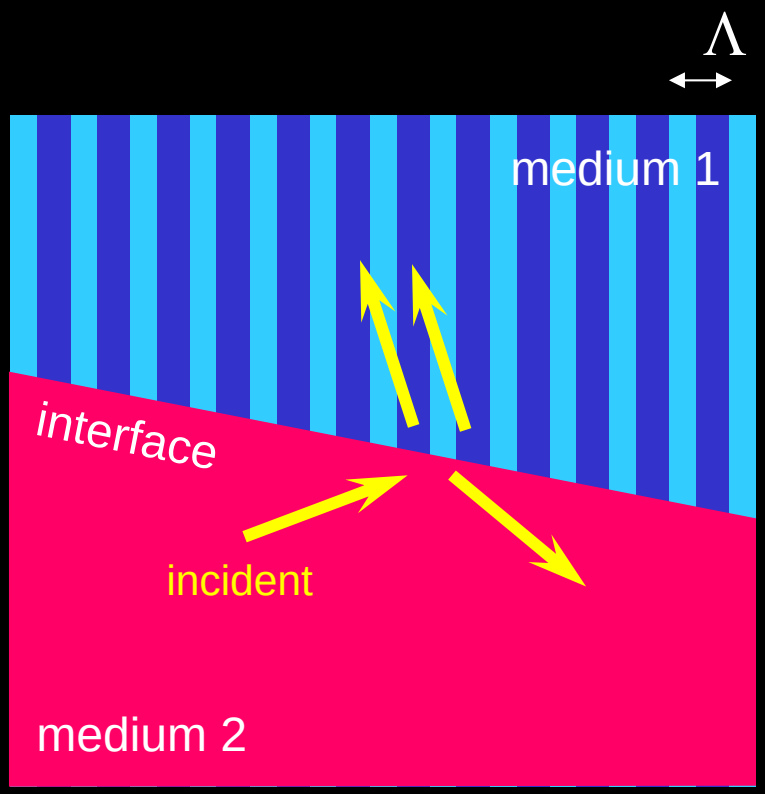
Bloch wave refraction and reflection

Mono-periodic medium



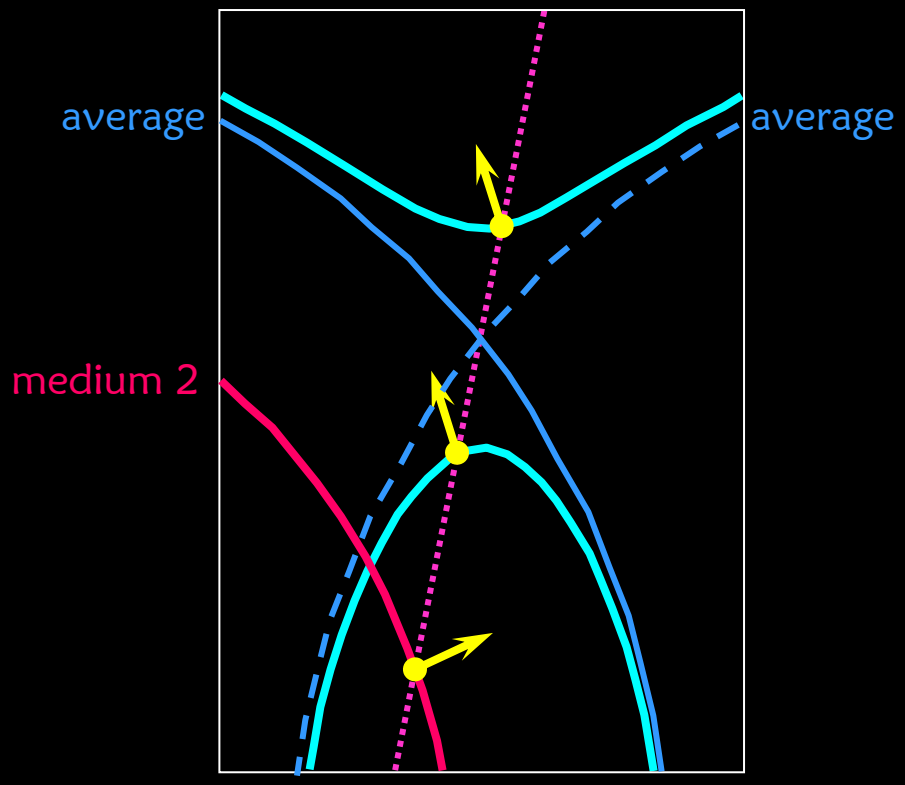
wavevector diagram



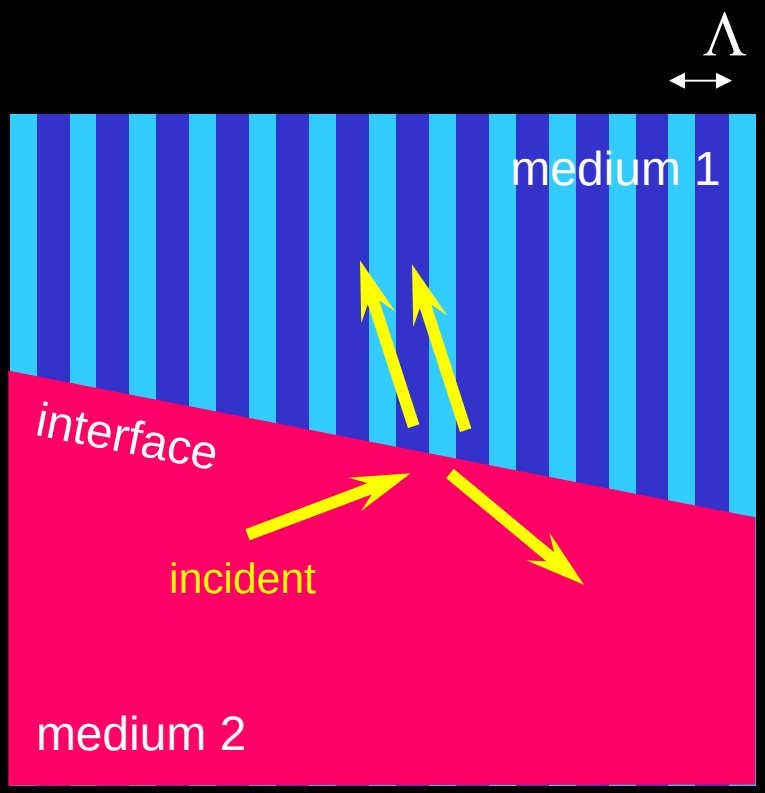
ray diagram

Double negative refraction

$$\mathbf{V}_{\text{group}} = \nabla_{\mathbf{k}}\omega(\mathbf{k})$$



wavevector diagram



Negative & positive refraction

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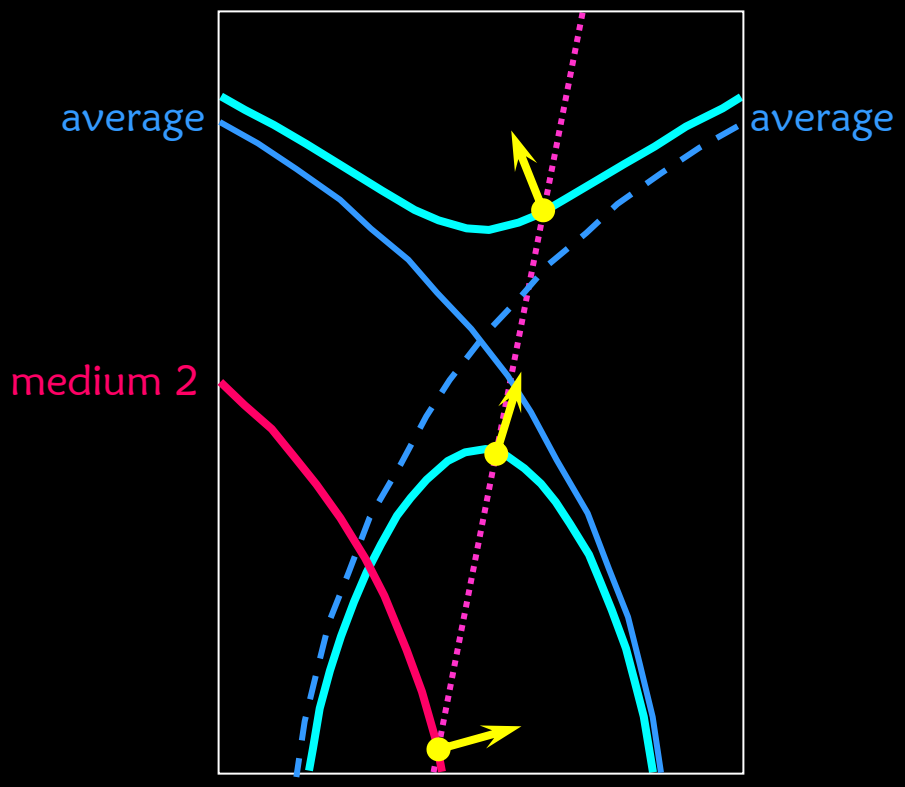
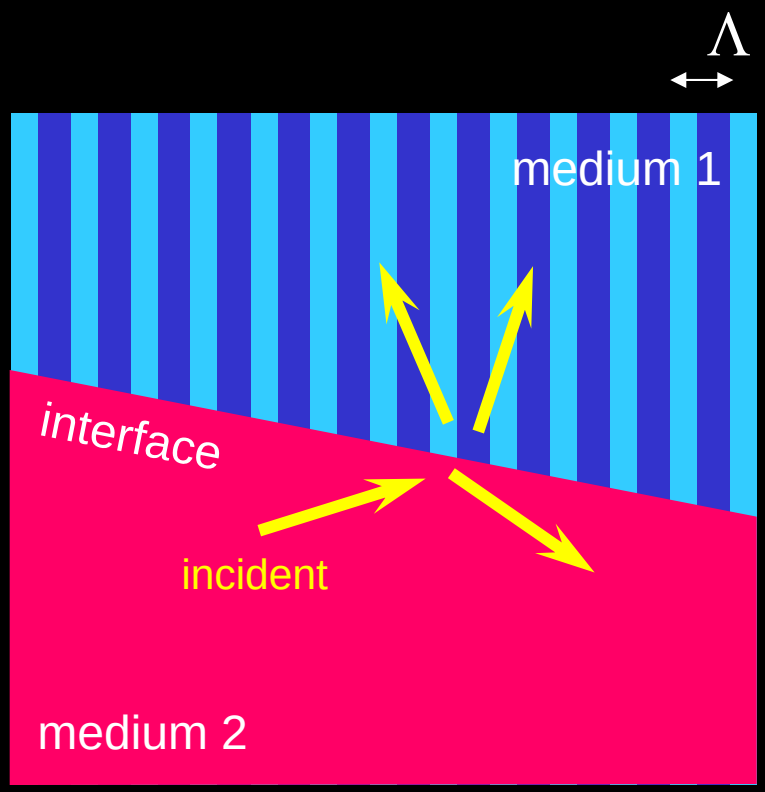


wavevector diagram



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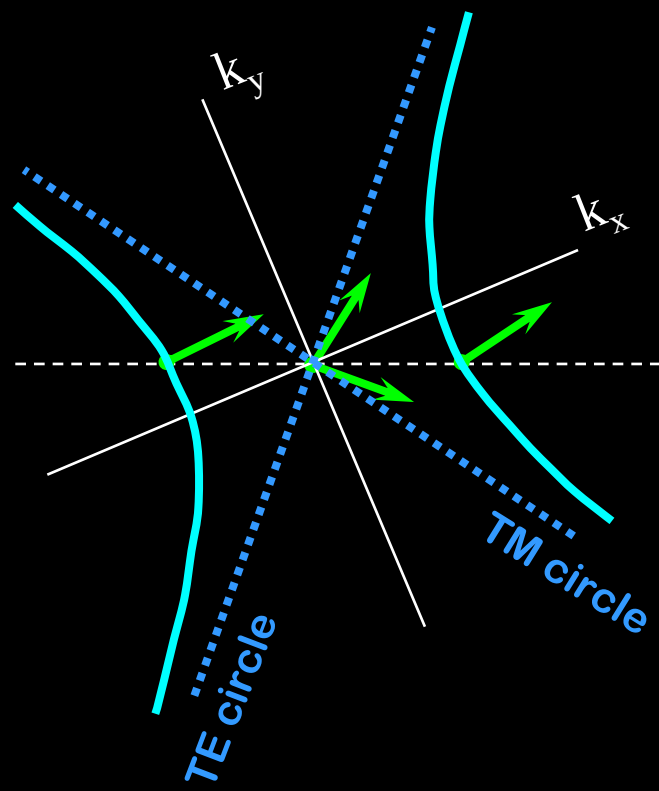
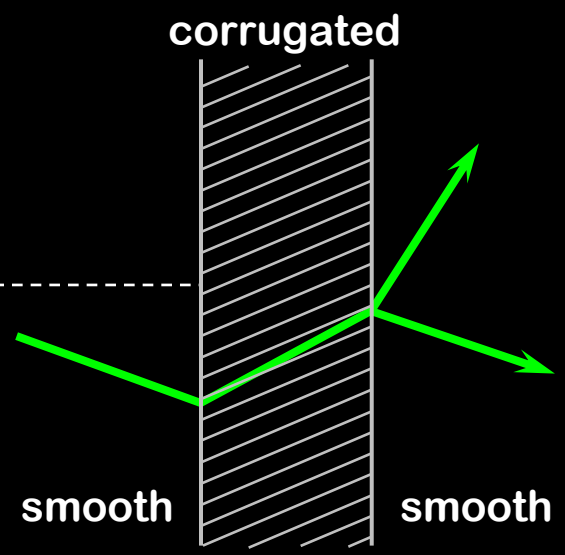
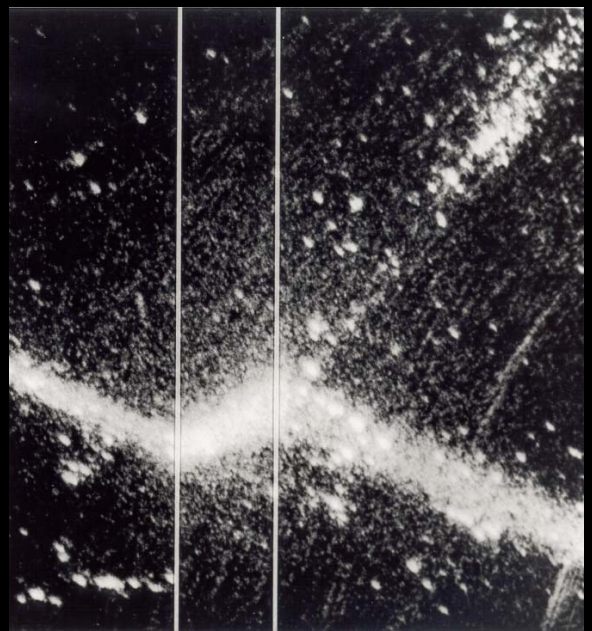
Negative refraction (1983)





- 150 nm sputtered tantala on borosilicate glass
- ~1 dB/cm losses
- enhanced scattering in periodic region

in "Confined Electrons and Photons" (Eds Burstein & Weisbuch) pp 585-633 Plenum Press, 1995



For reprints see:
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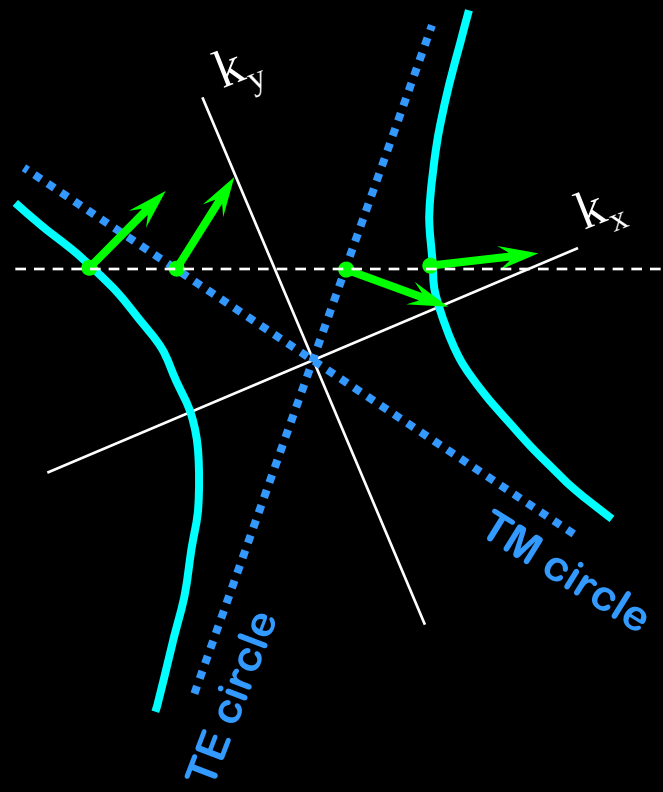
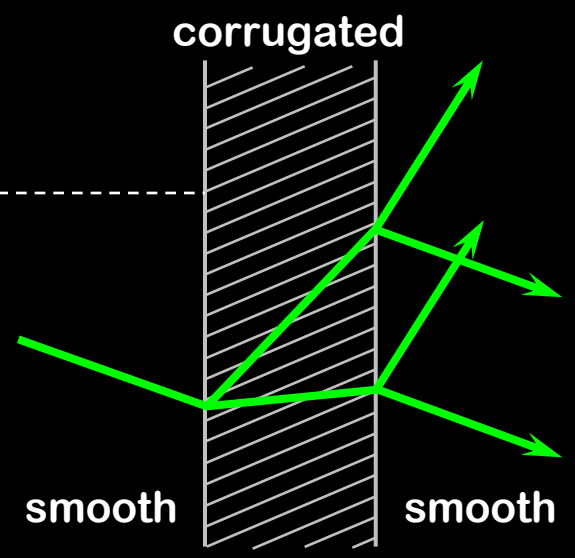
Double negative refraction (1983)





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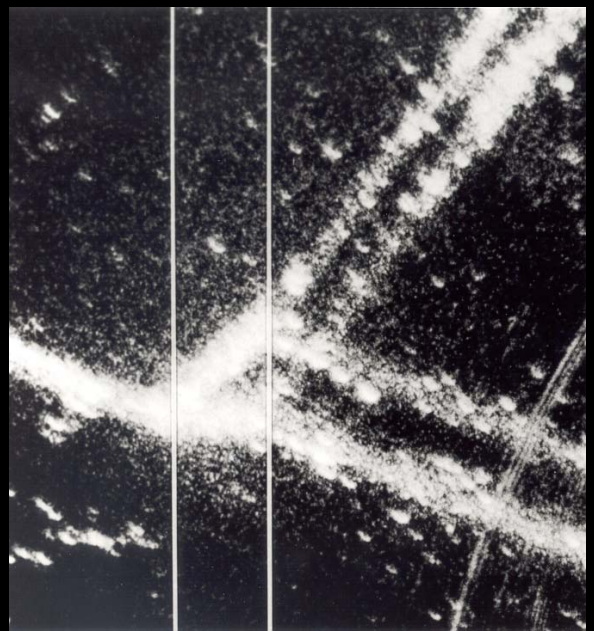

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Double negative refraction

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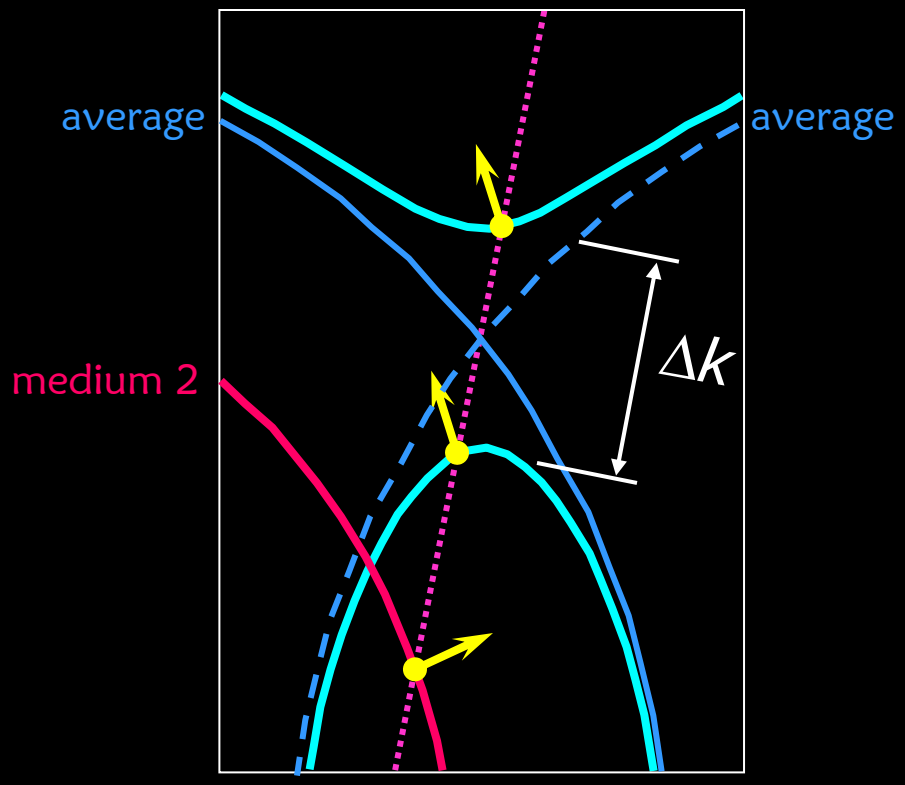
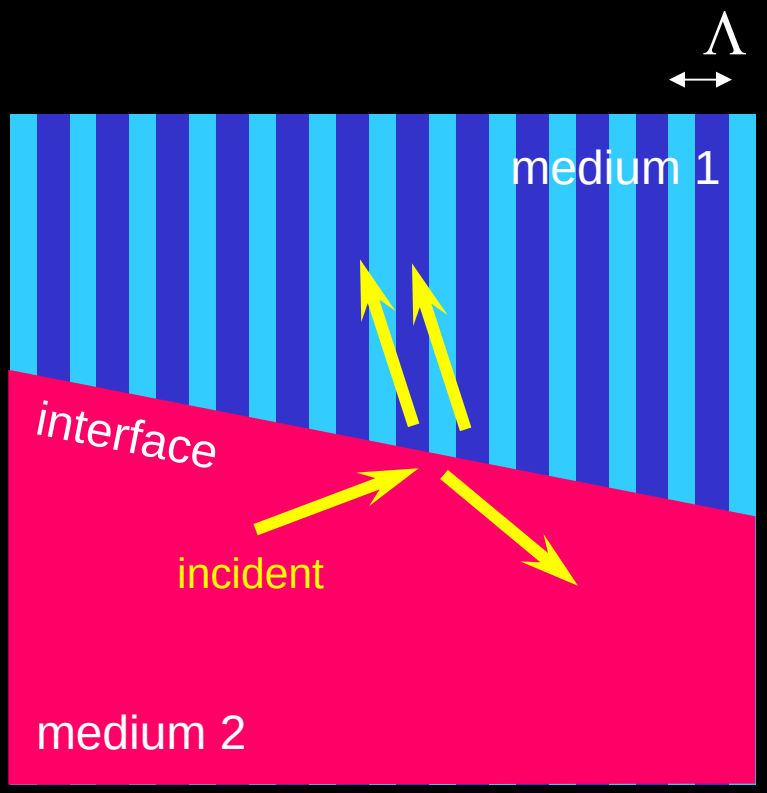


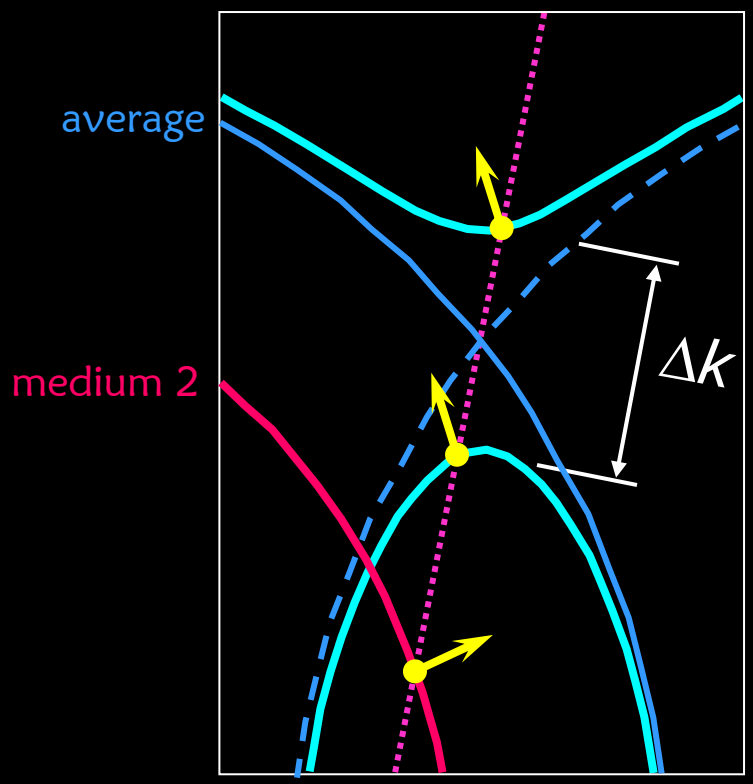
wavevector diagram



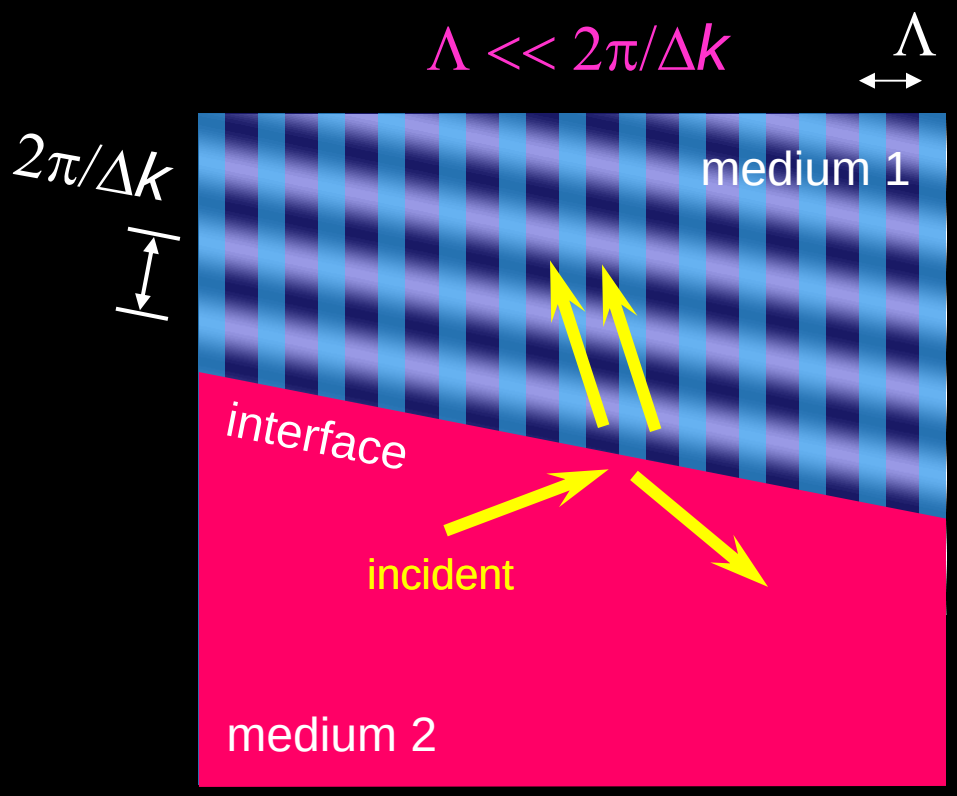
ray diagram

Bloch wave interference

Phys Rev **A33** (3232-3242) 1986

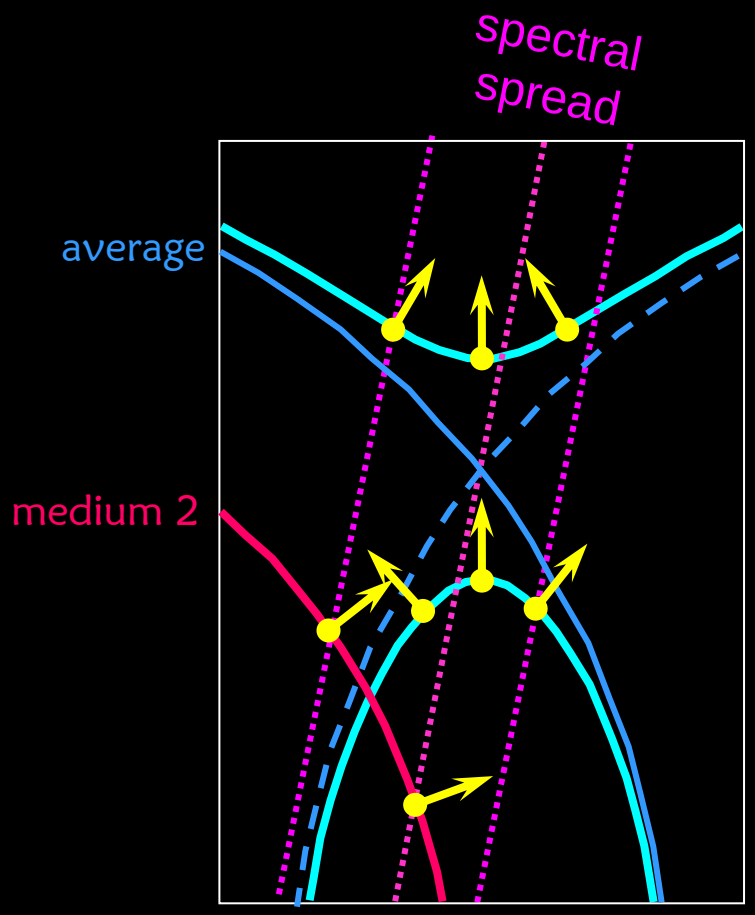


wavevector diagram

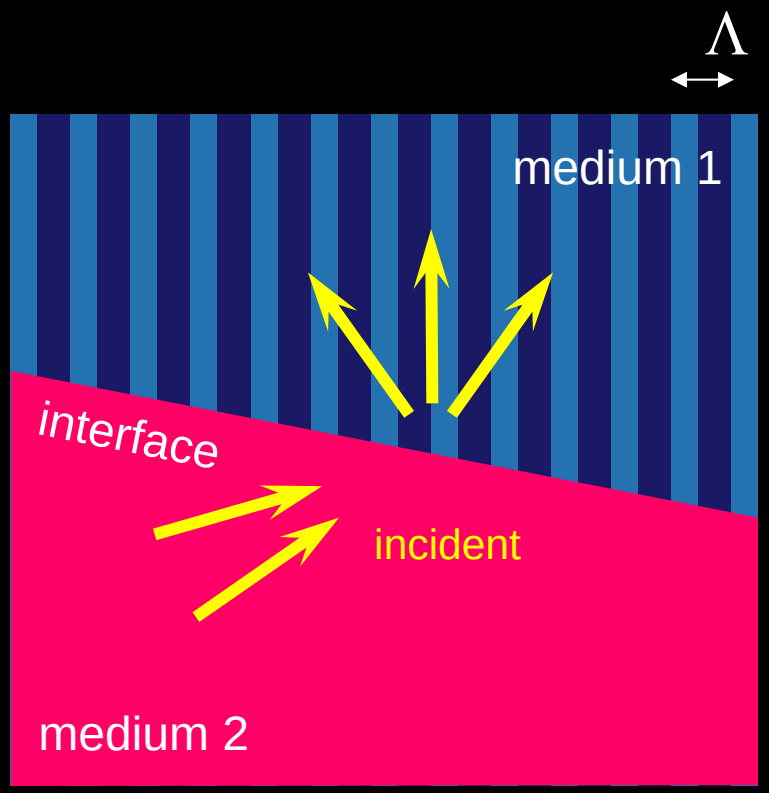


ray diagram

Bloch wave point-influence function



wavevector diagram



ray diagram

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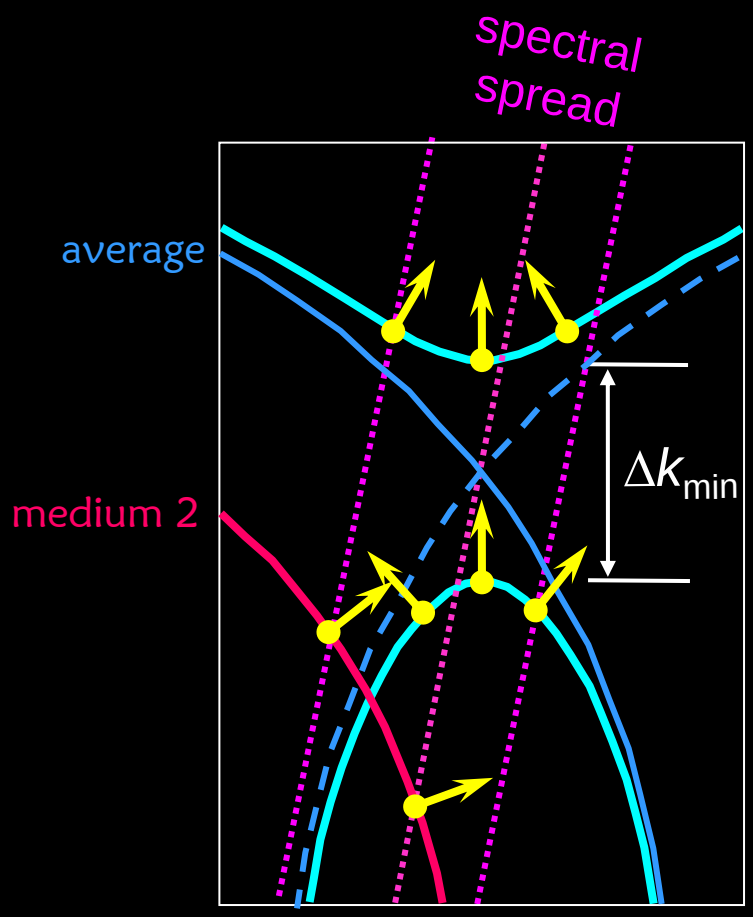
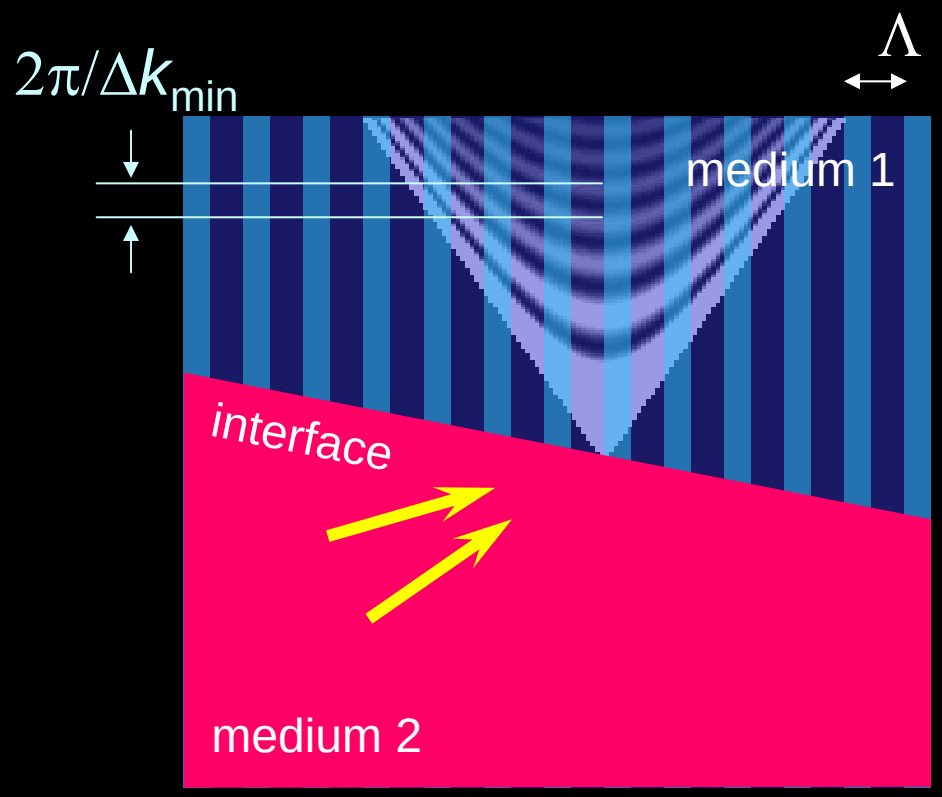


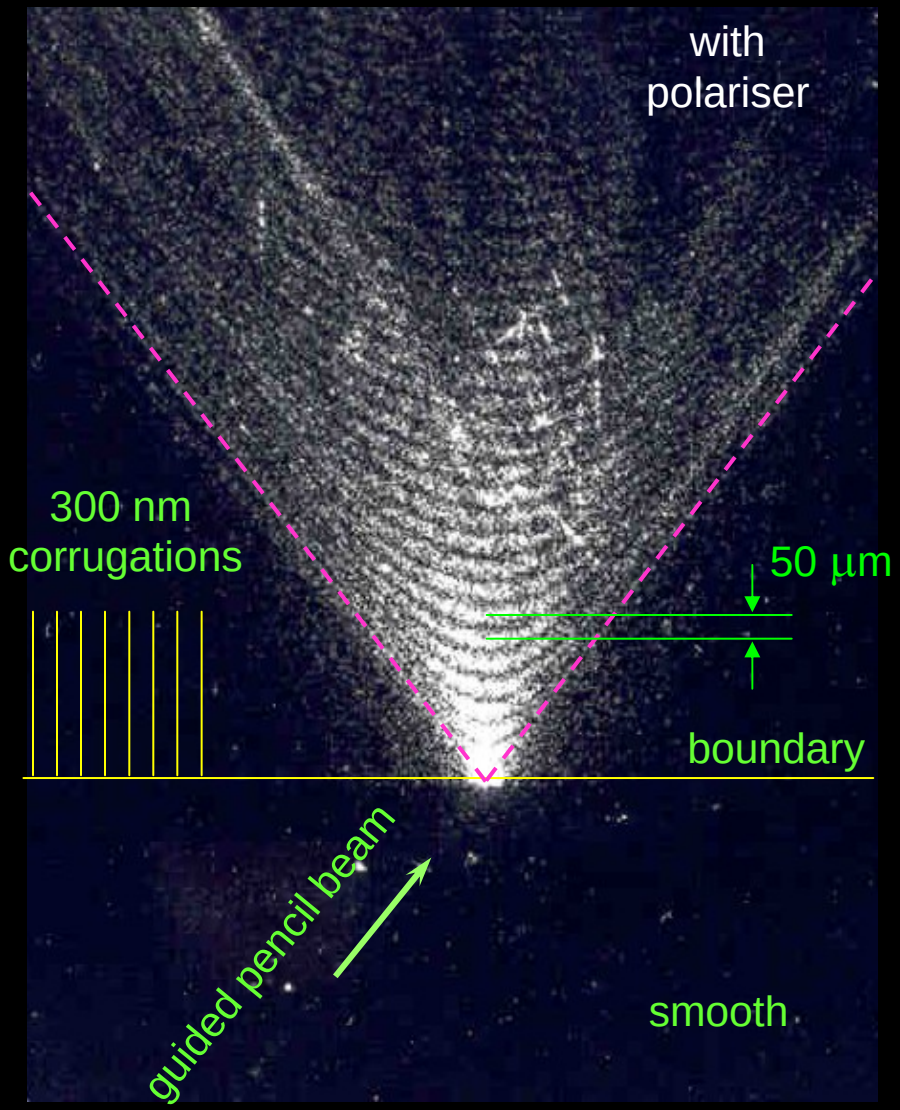
wavevector diagram



ray diagram

Phys Rev **A33** (3232-3242) 1986

Experimental observation



- **Pendellösung period 50 µm**
- **stop-band width 125 per mm**
- **100% directional coupler is only 25 µm thick**

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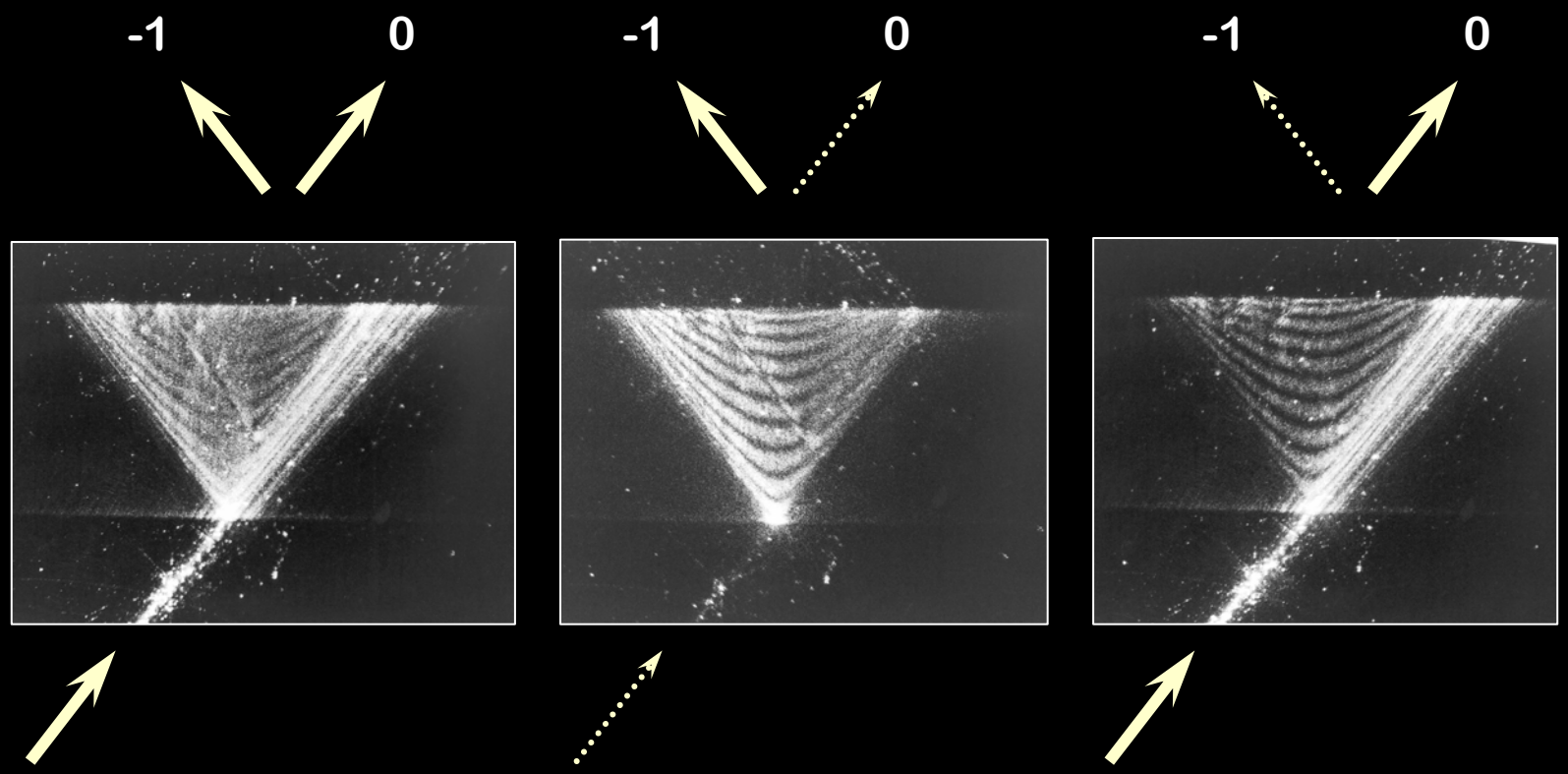
Experimental observations

Phys Rev **A33** (3232-3242) 1986



no polarizer

"0 " wave blocked with polarizer

"-1 " wave blocked with polarizer

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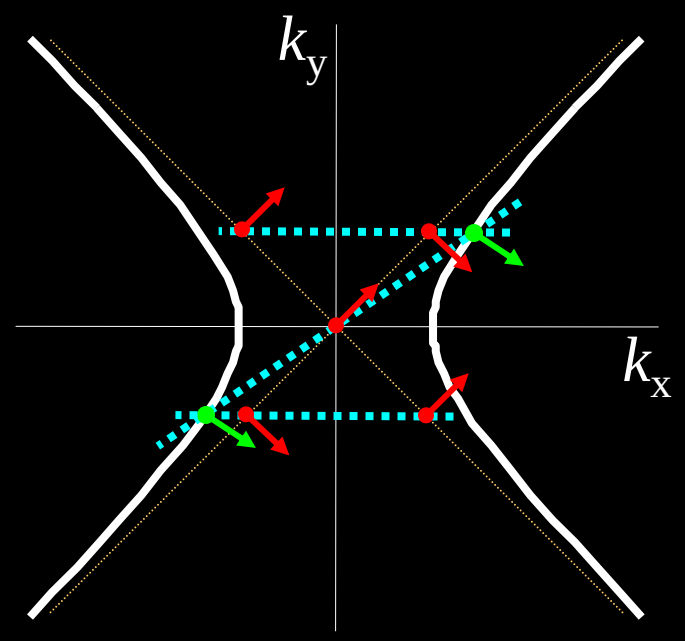
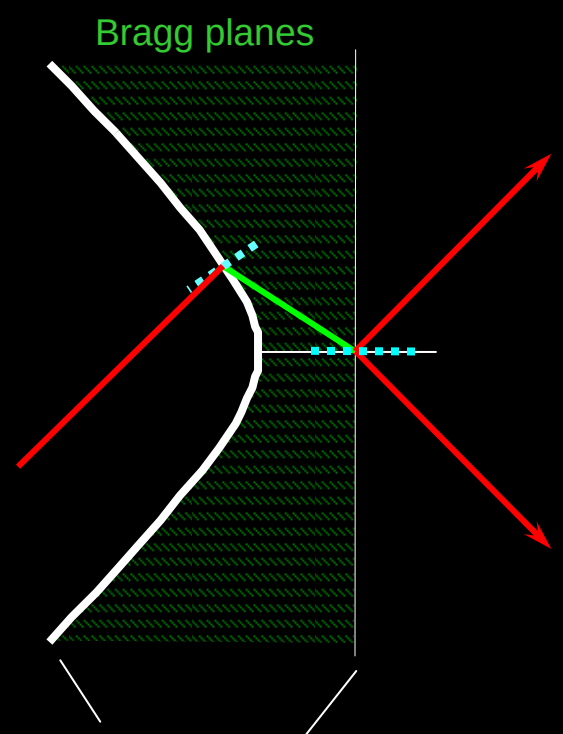
Bloch wave lens

using negative and positive refraction



wavevector diagram near Bragg point



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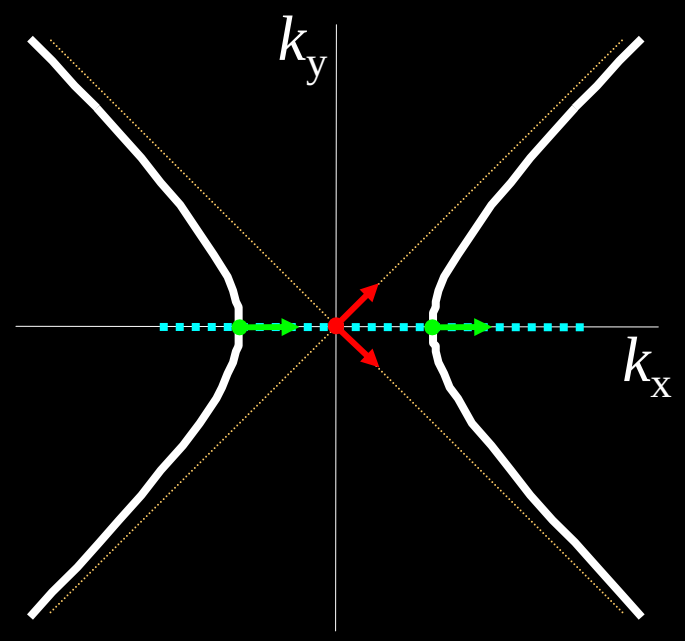

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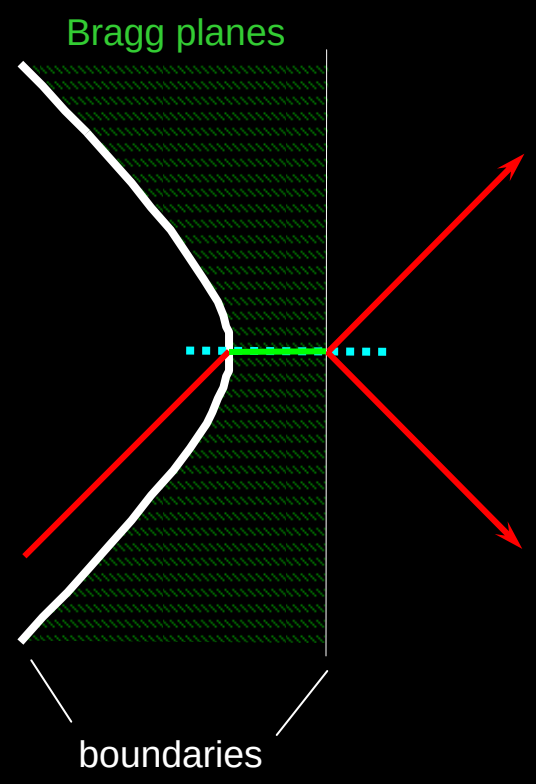
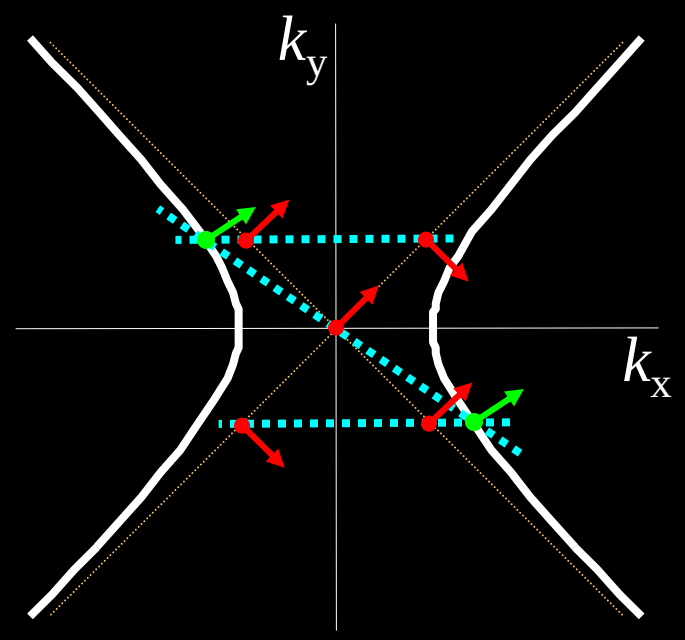


Bloch wave lens

using negative and positive refraction



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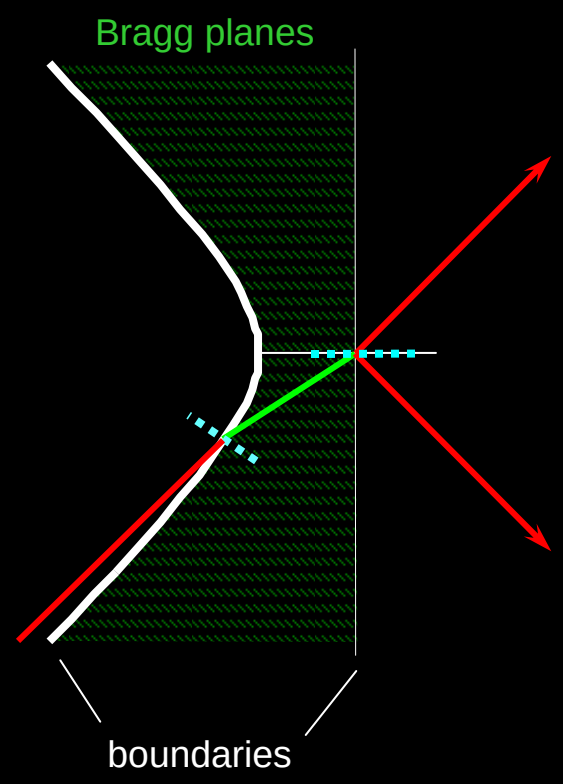
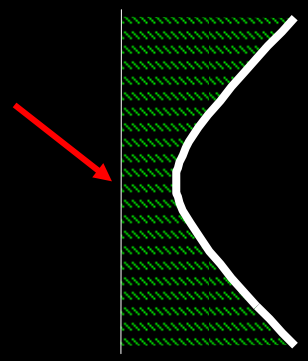
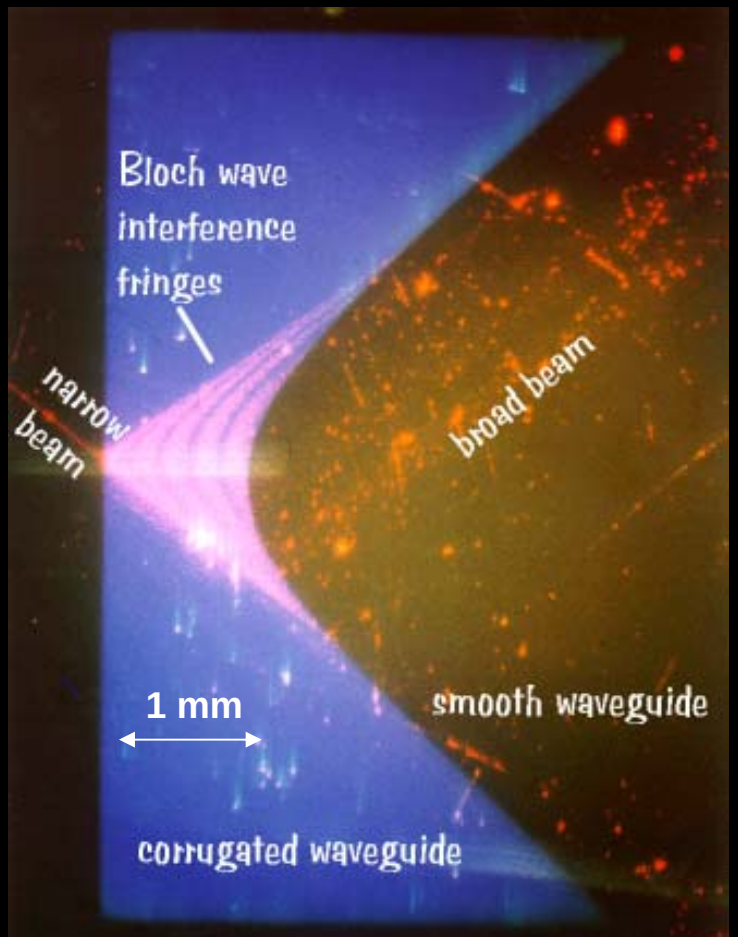


Bloch wave beam expander

Electron. Lett., 20 (72-73) 1984

- miniature optical elements
  - two-dimensional resonators
  - in-plane 2D lasers
  - new kinds of lenses





Bloch wave beam expander

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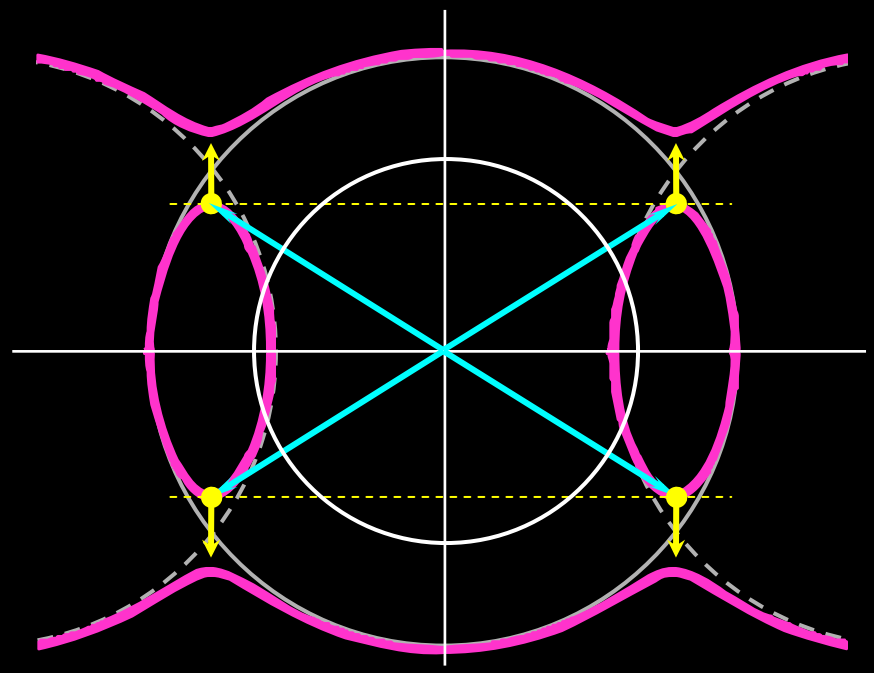
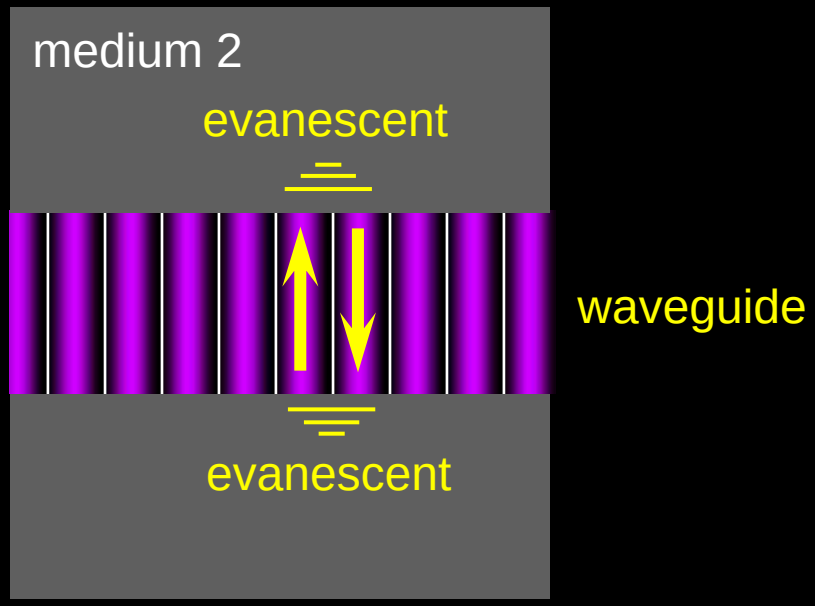
Total internal reflection at normal incidence

$$\mathbf{v}_g = \nabla_{\mathbf{k}} \omega(\mathbf{k}_B)$$





Erice 1993

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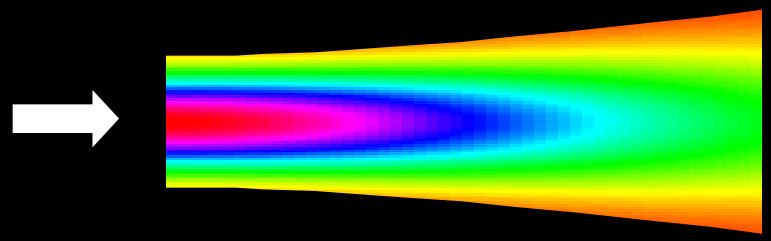
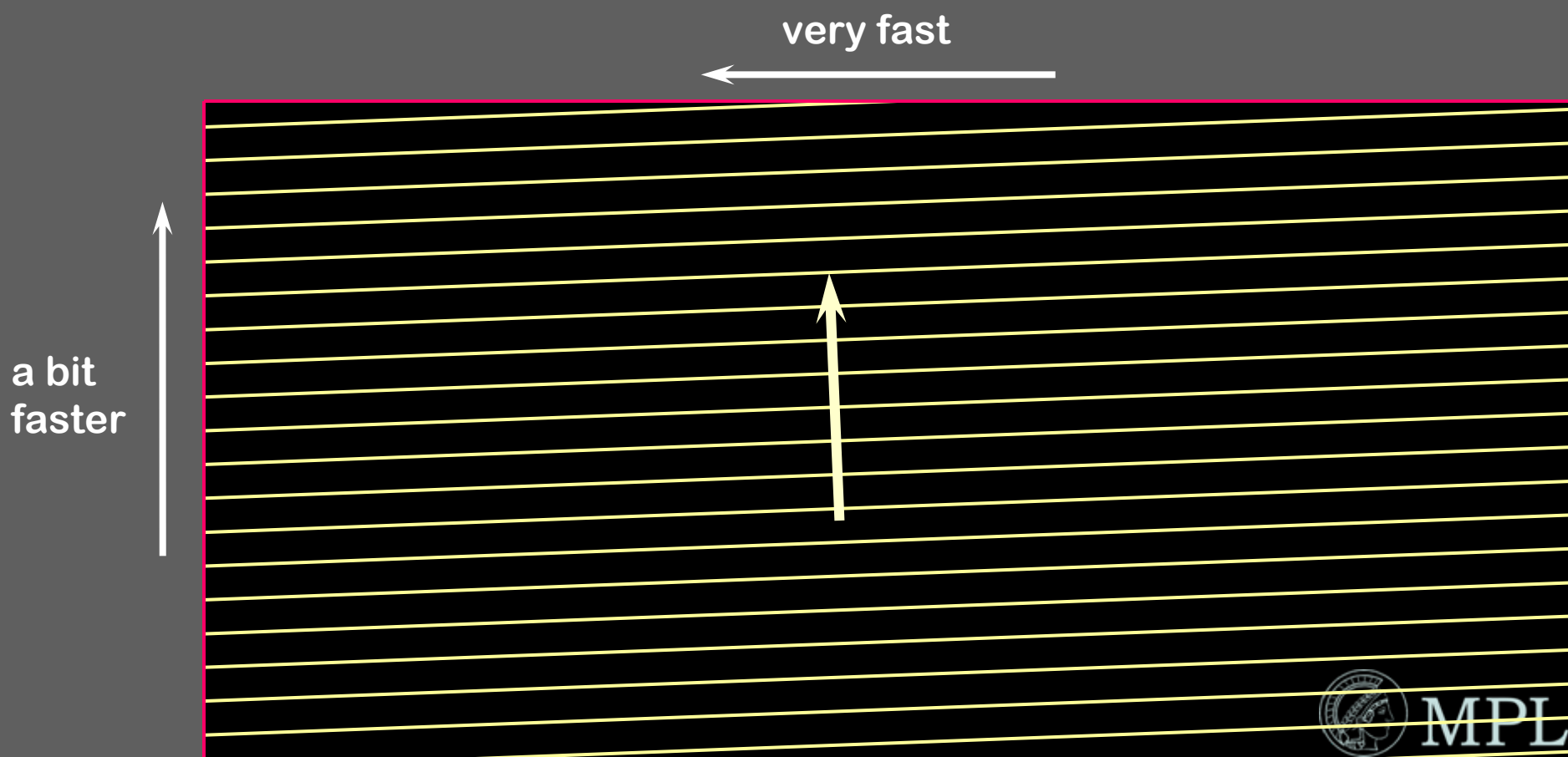
Diffraction (spatial dispersion)

diffraction wanted?



- **Free-space:** diffraction is OK but rather limited & isotropic
- **Photonic crystals:** diffraction can be very strong and anisotropic with multiple beams

Phase velocity is not a vector



"Slowness" vector

wavevector

$$\frac{c\,\mathbf{k}(\omega)}{\omega} = \mathbf{n}(\omega)$$

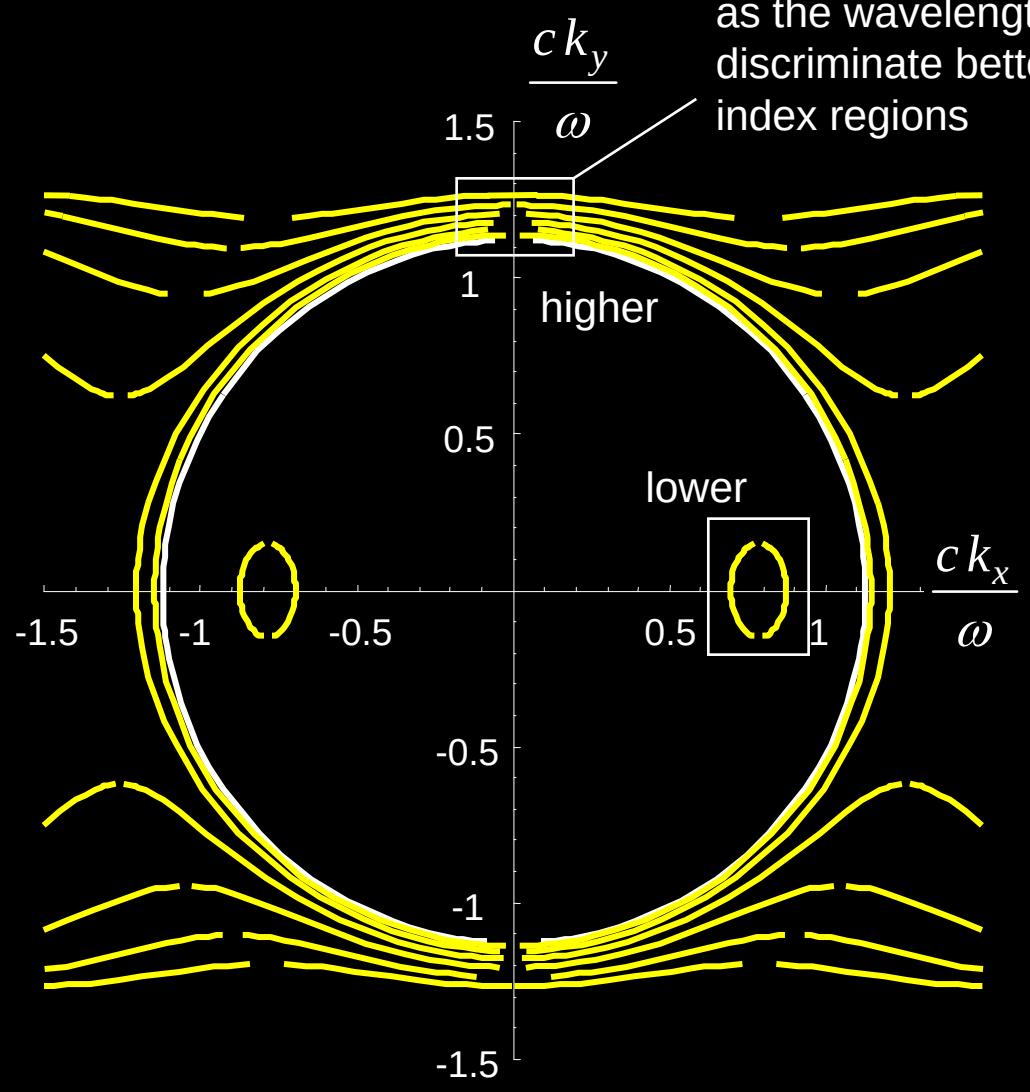
frequency

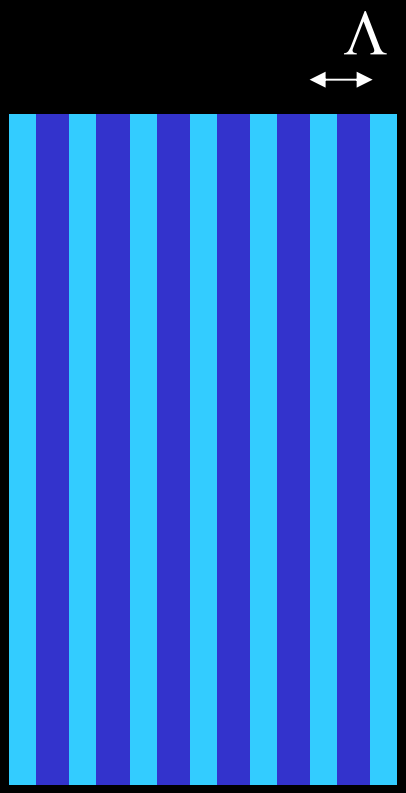
vector refractive index

- **highlights the change in refractive index with frequency and direction**

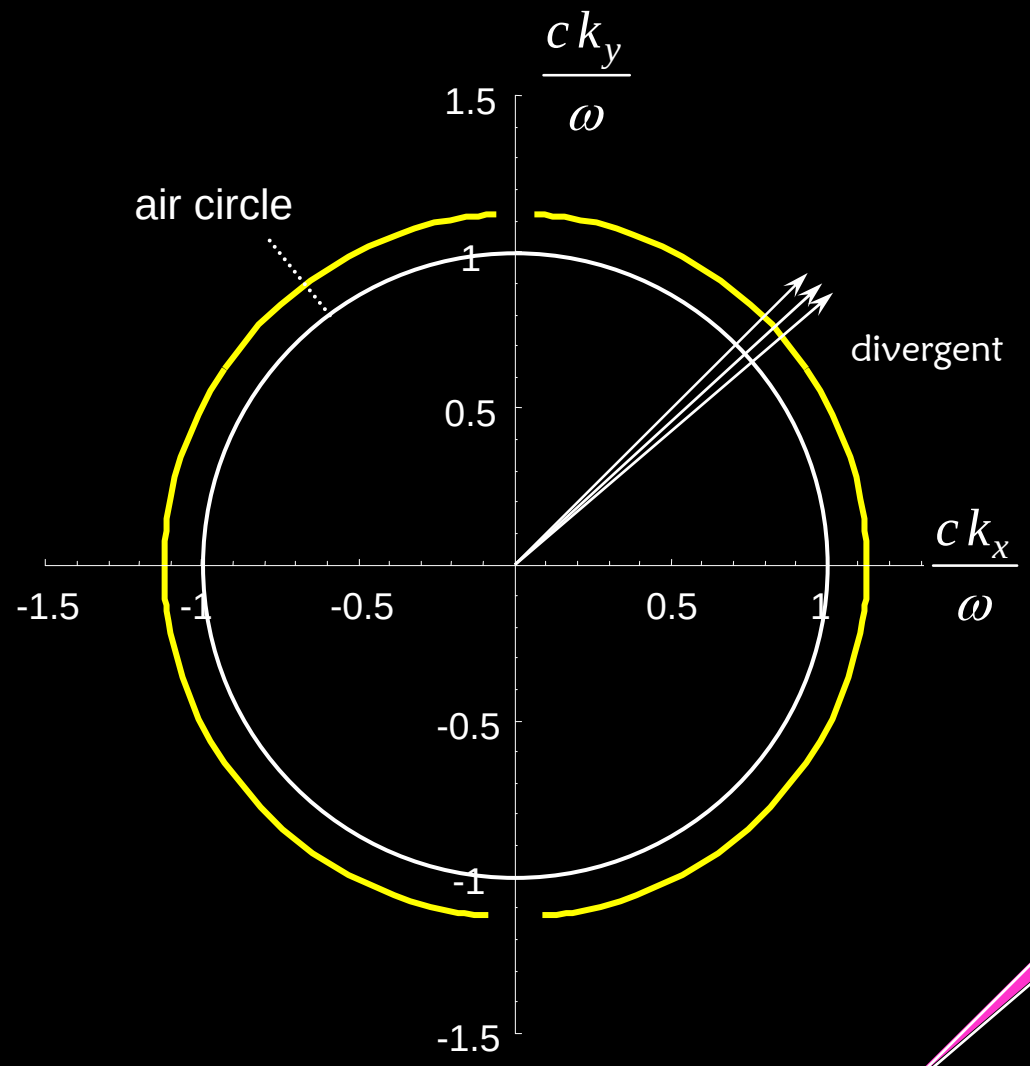
Slowness diagrams for light: TE

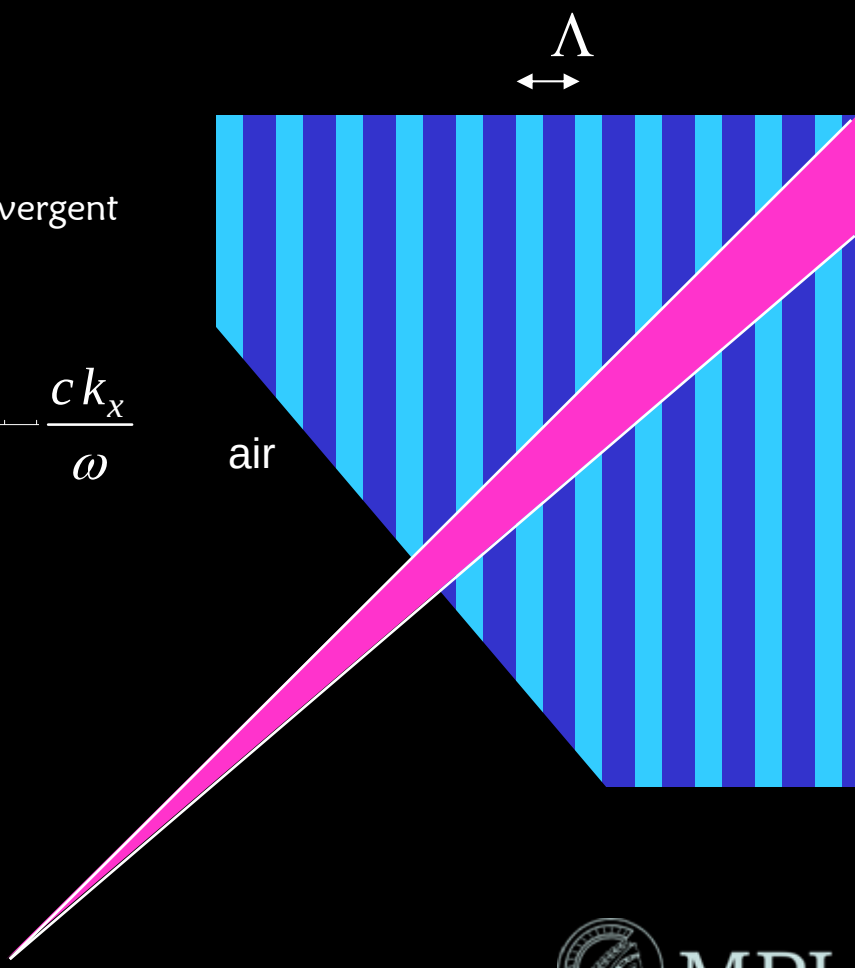
as the wavelength drops, the fields can discriminate better between low & high index regions



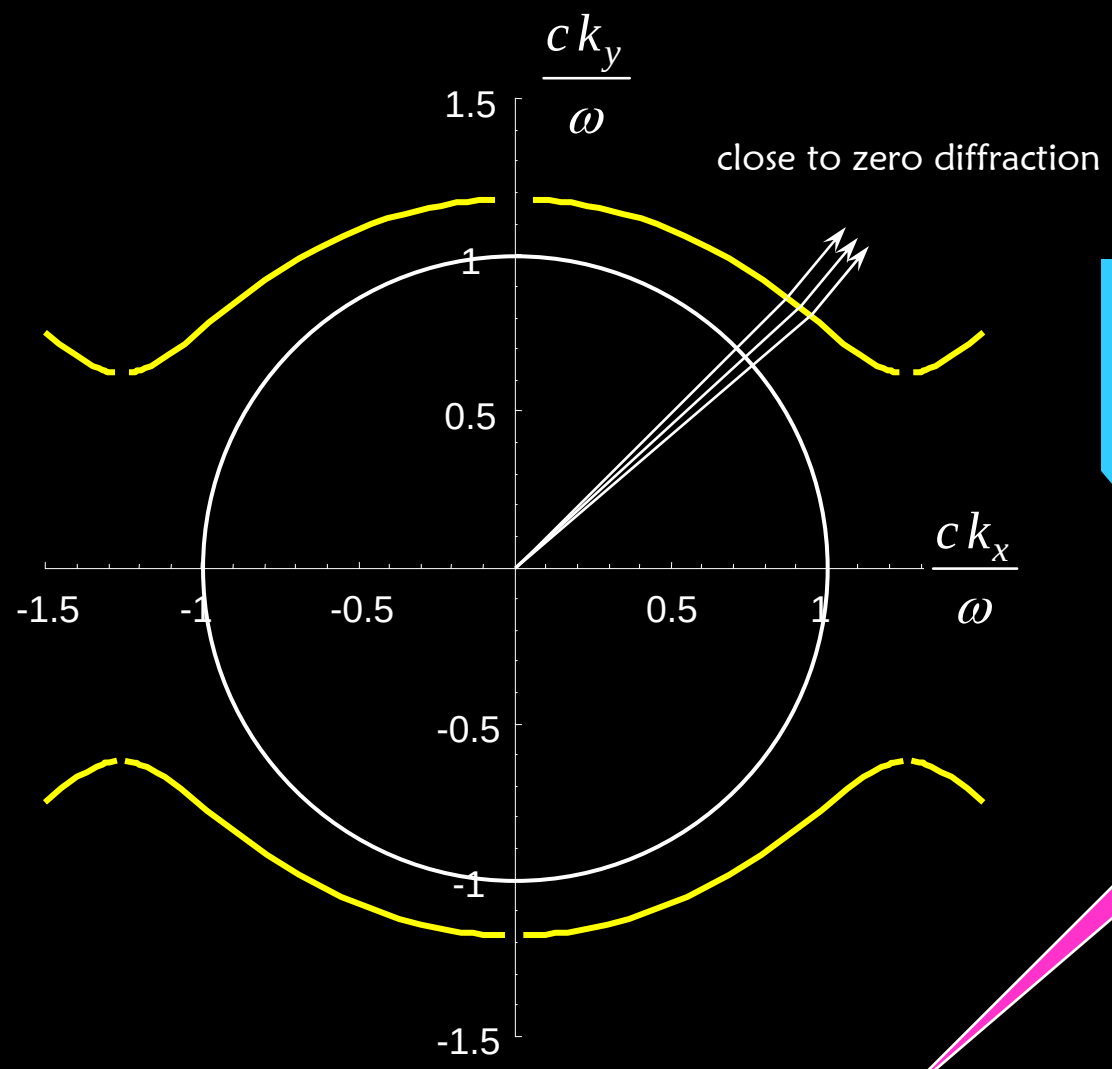


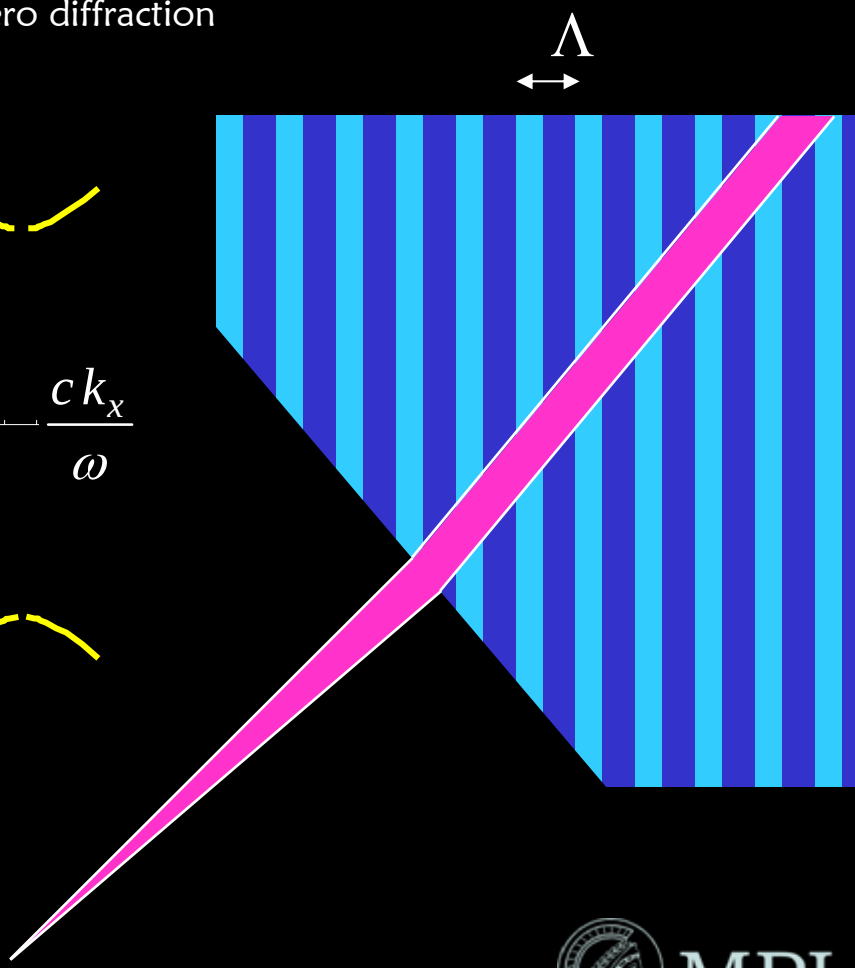
Slowness diagrams: Beam diffraction



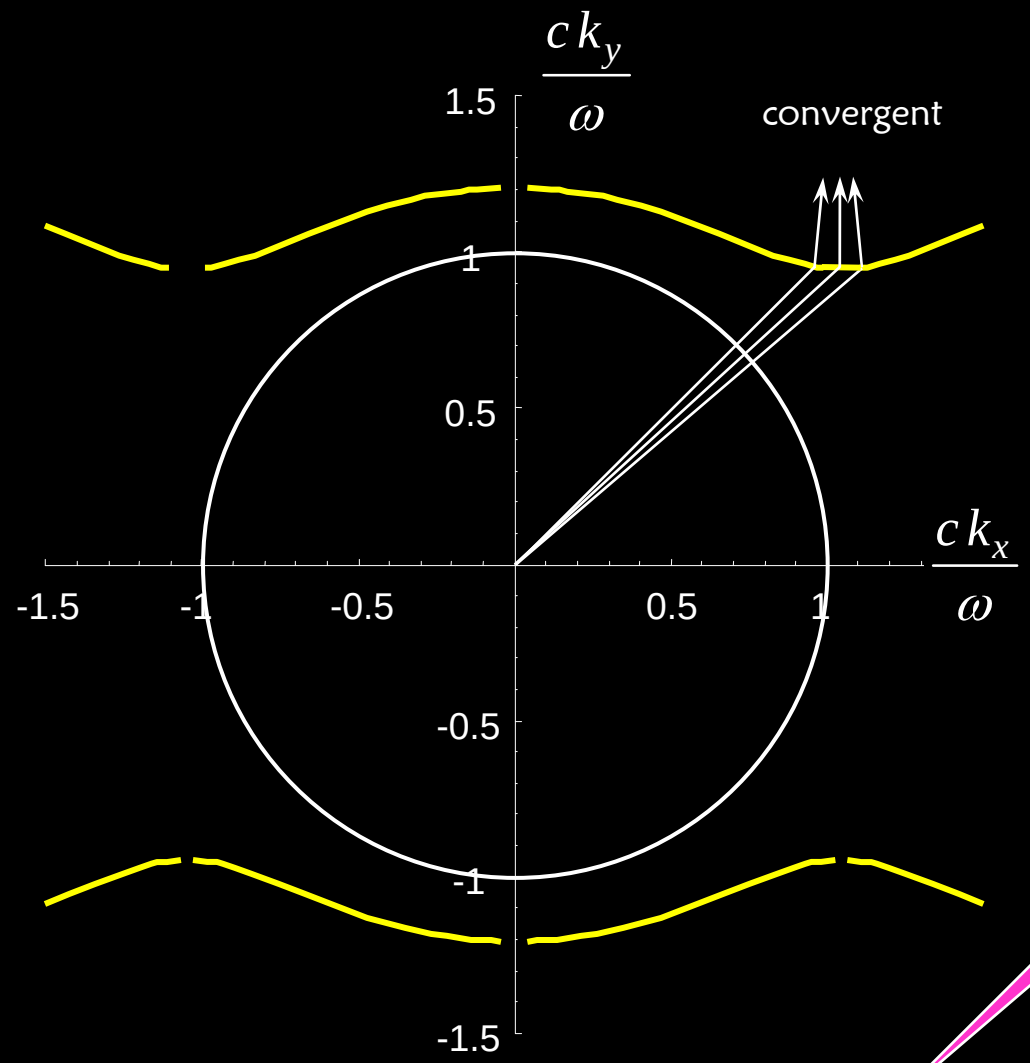


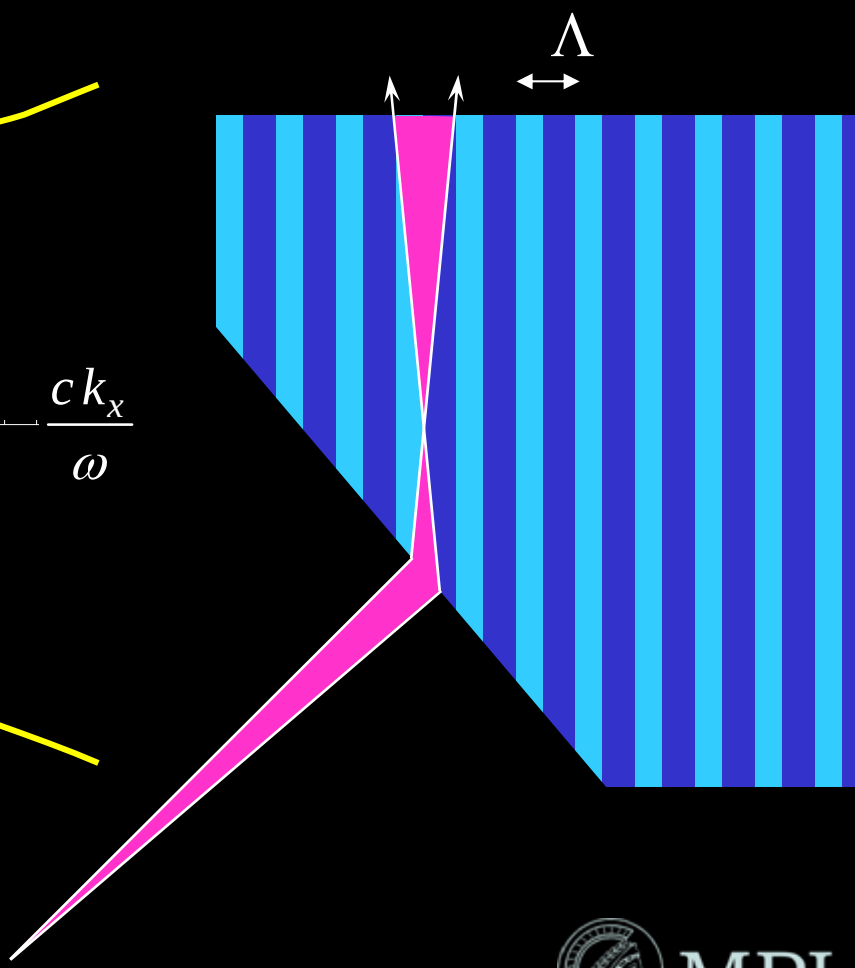
Slowness diagrams: Beam diffraction



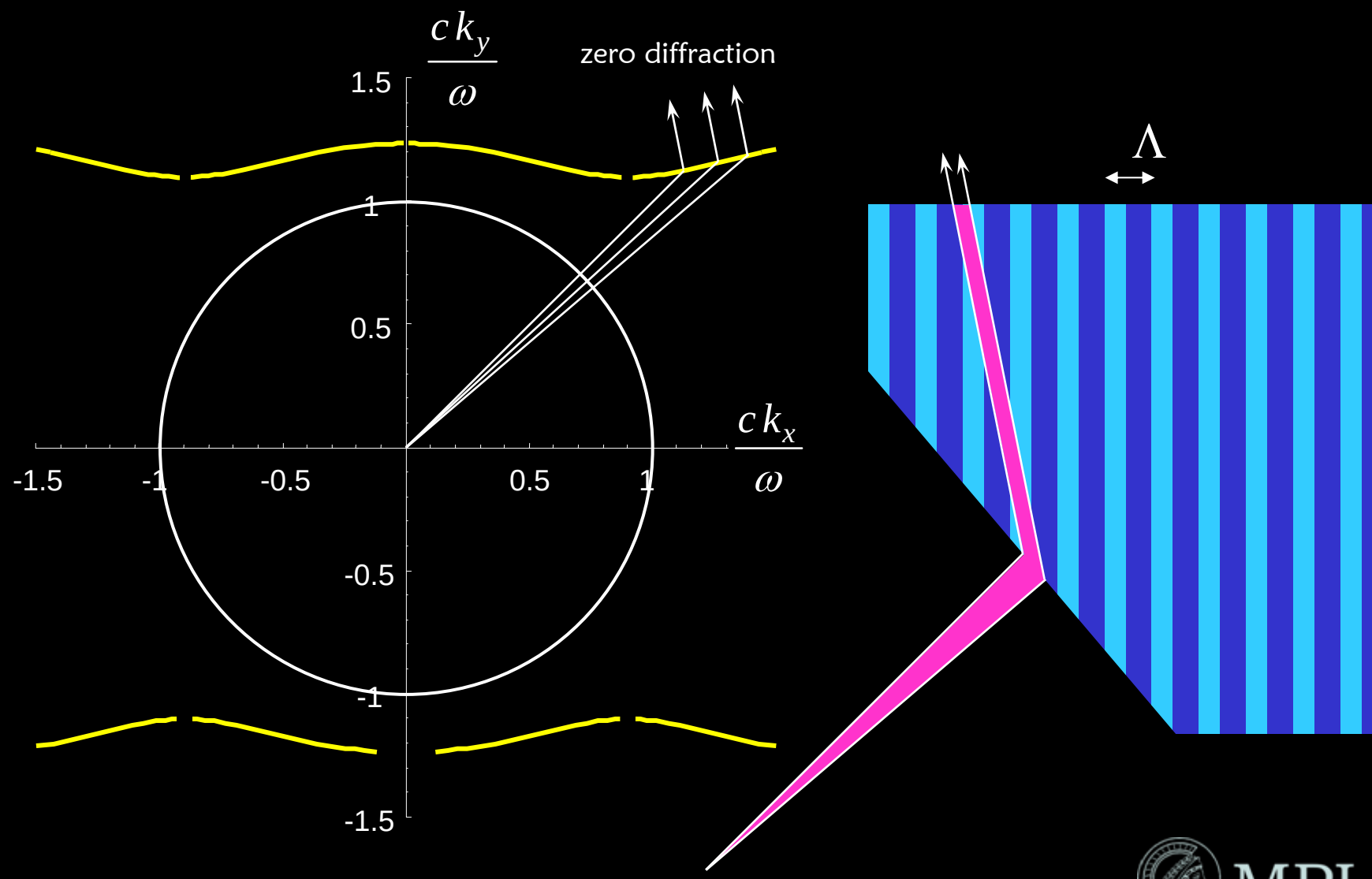


Slowness diagrams: Beam diffraction

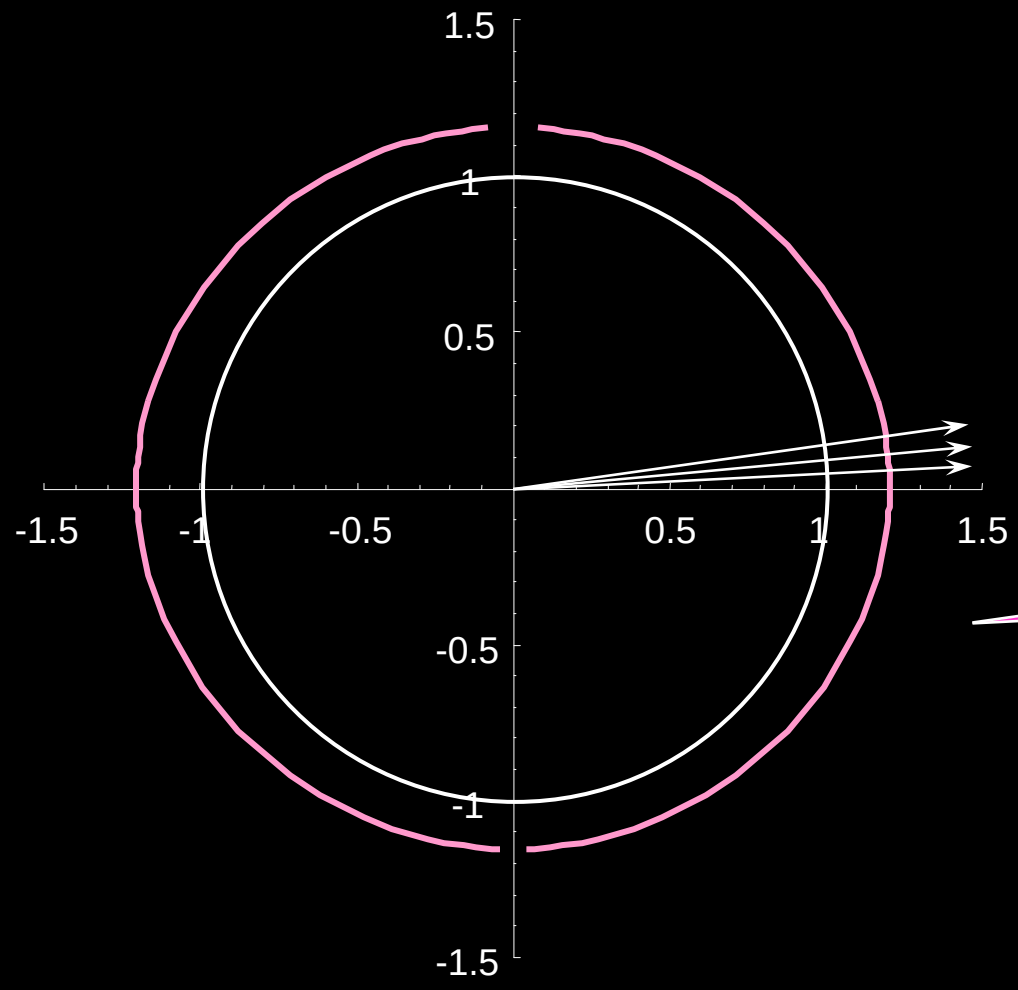


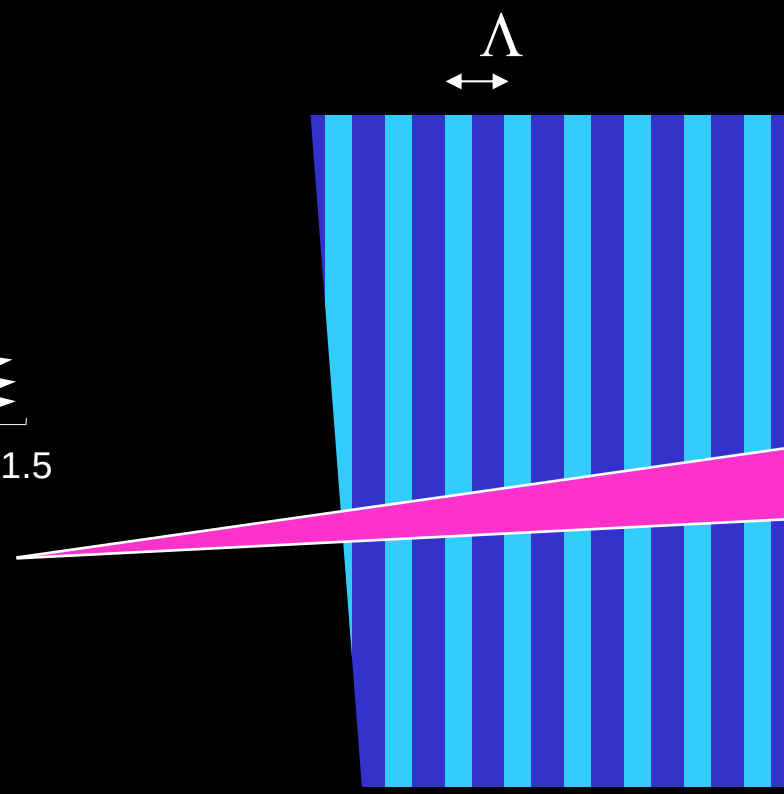


Slowness diagrams: Beam diffraction

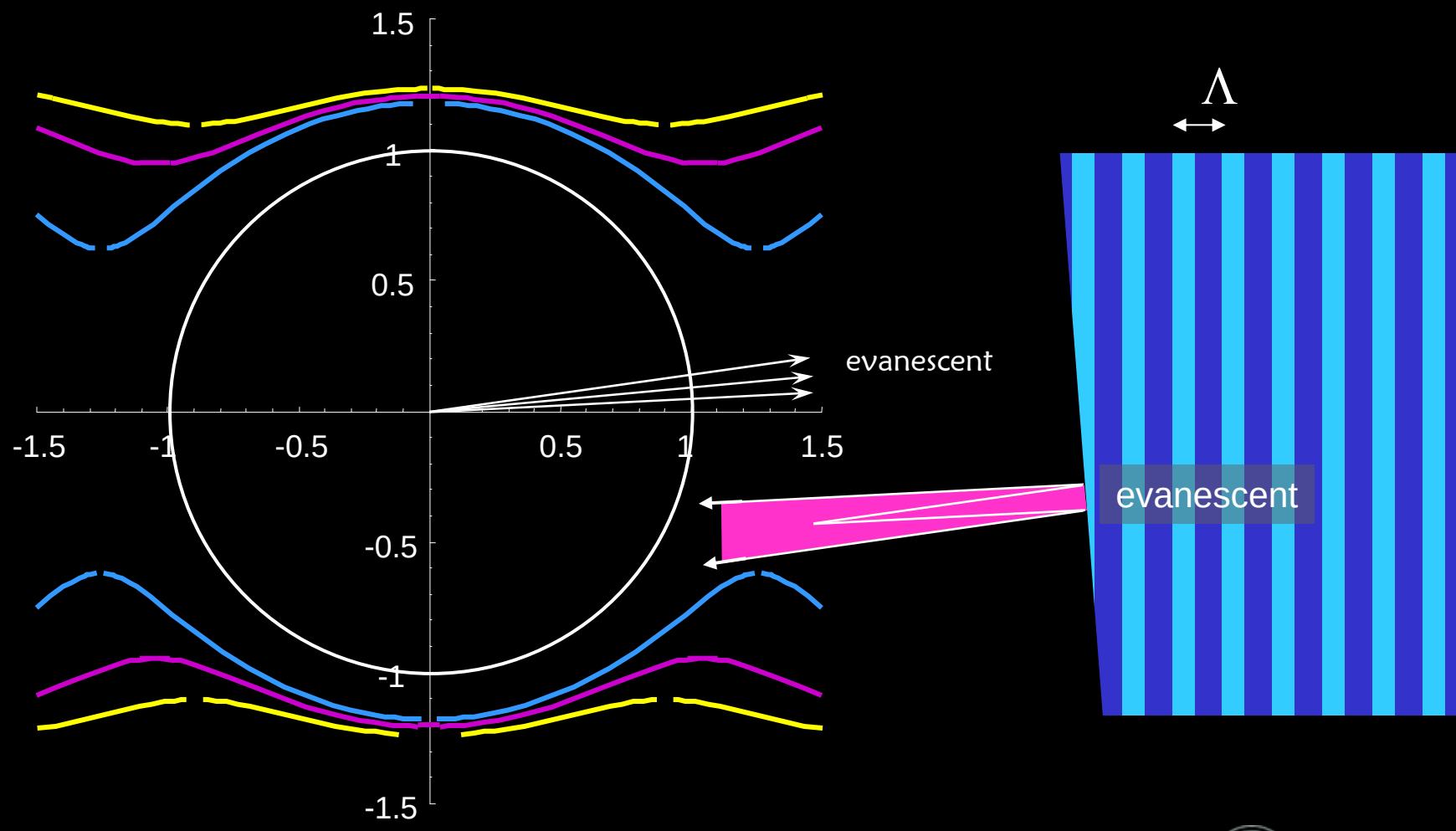


Slowness diagrams: Beam diffraction

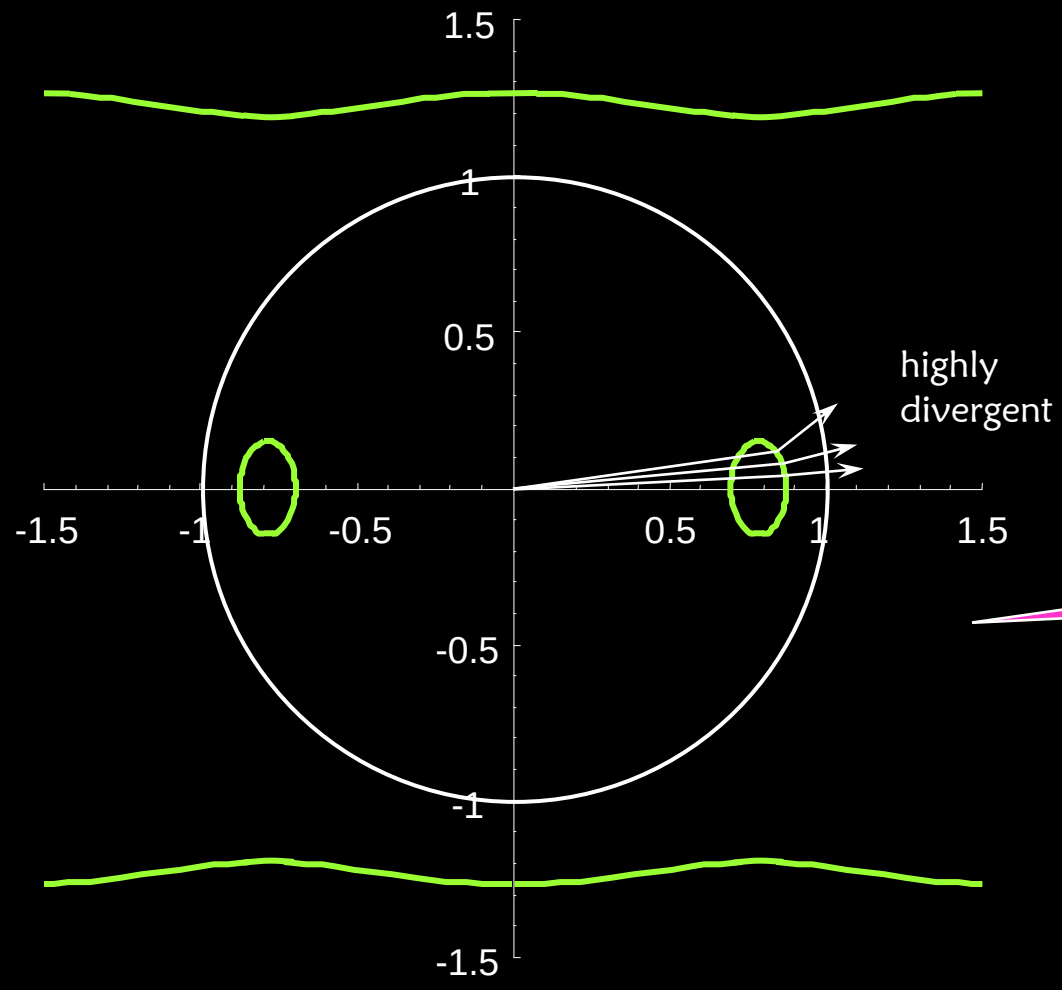


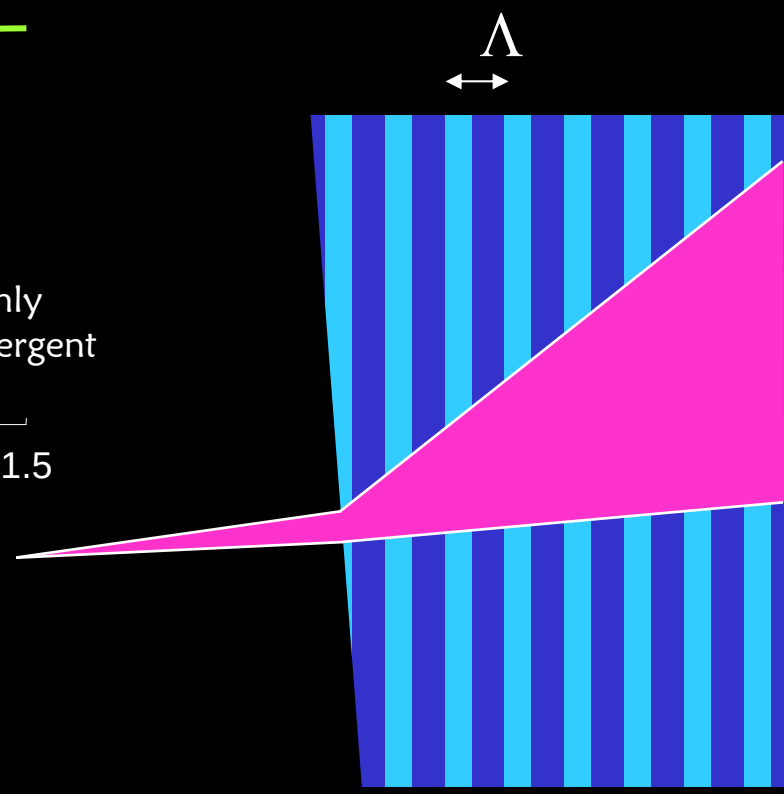


Slowness diagrams: Beam diffraction



Slowness diagrams: Beam diffraction





Topics

- Peculiar Bloch waves
  - nearly free photon model
  - wavevector diagrams
  - anatomy of a Bloch wave
  - negative & positive refraction
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  - TIR at normal incidence
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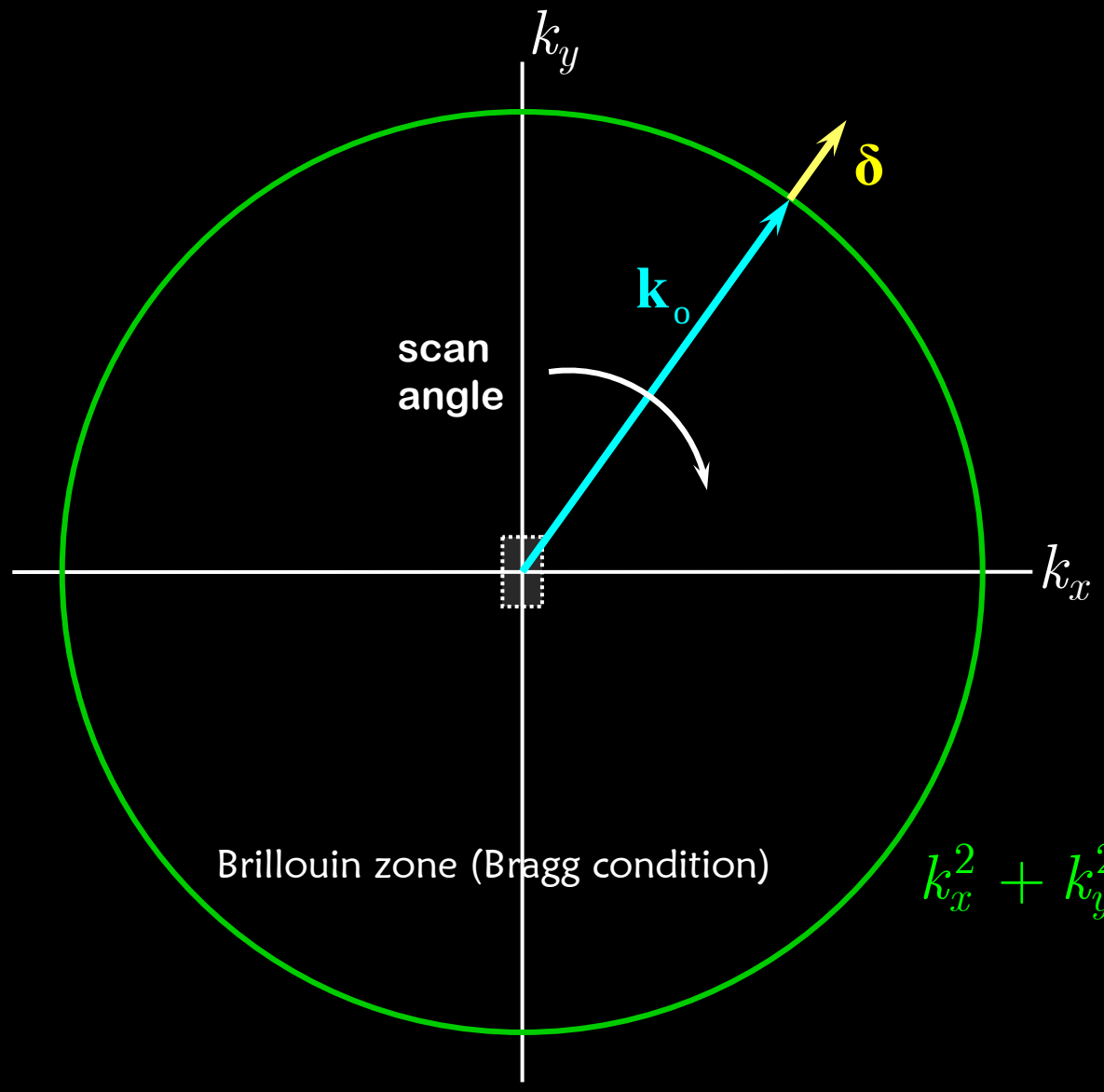
  - square lattices
- Conclusions

Square lattice



$$k_x^2 + k_y^2 = n_o^2 k^2$$

Nearly-free photons in square lattice

$$\varepsilon/\varepsilon_o = 1 + M(\cos Kx + \cos Ky)$$

perturbation

$$E(x,y) = \sum_{m=0}^{\hat{m}} \sum_{n=0}^{\hat{n}} V_{mn} \exp - j\left[((k_o + \delta)\cos\theta - mK)x + ((k_o + \delta)\sin\theta - nK)y\right]$$

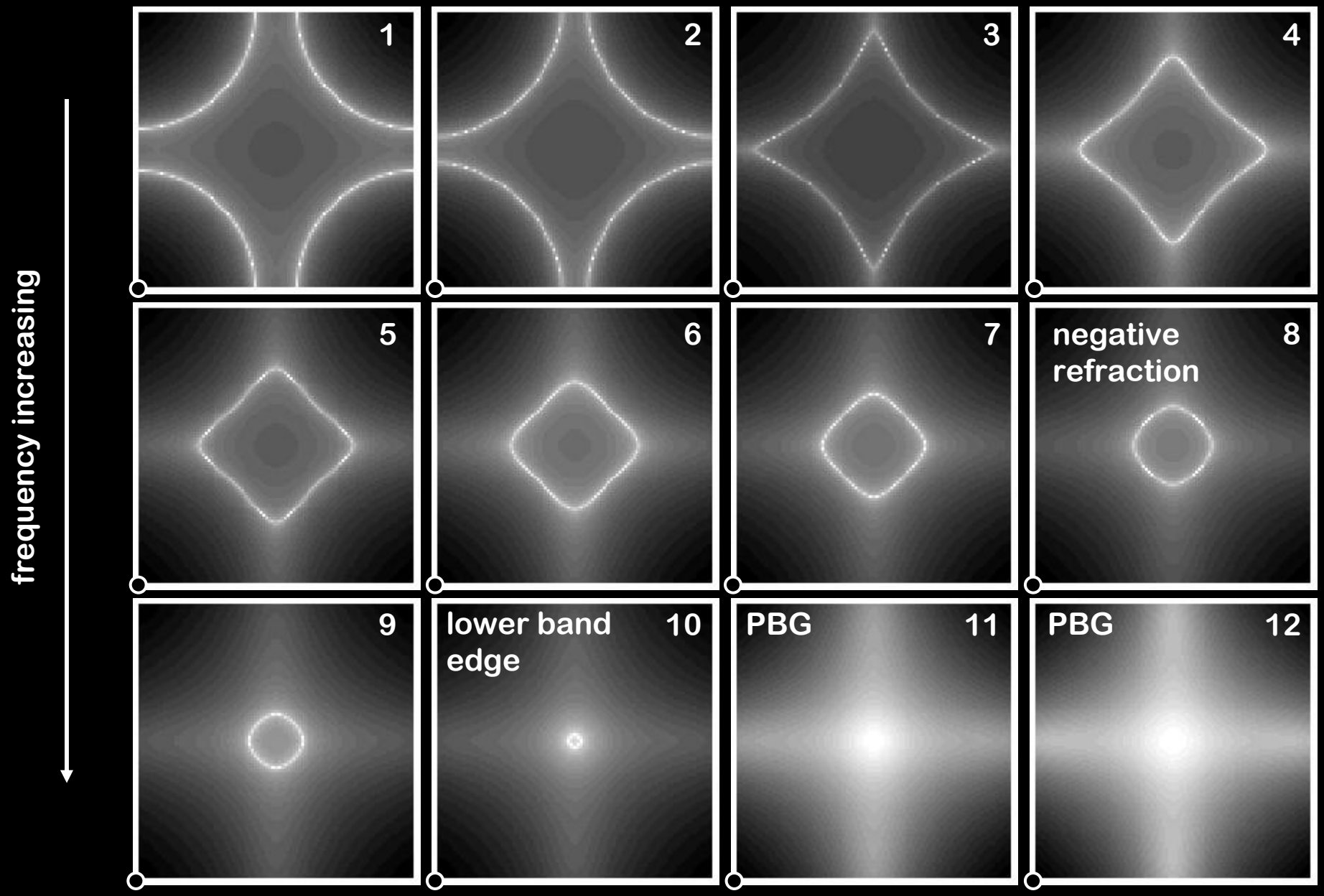
$$q = -\delta(\delta + 2k_o)$$

$$a_s = \frac{4\pi}{\Lambda}\left(\frac{\pi}{\Lambda} - (\delta + k_o)\sin\theta\right)$$

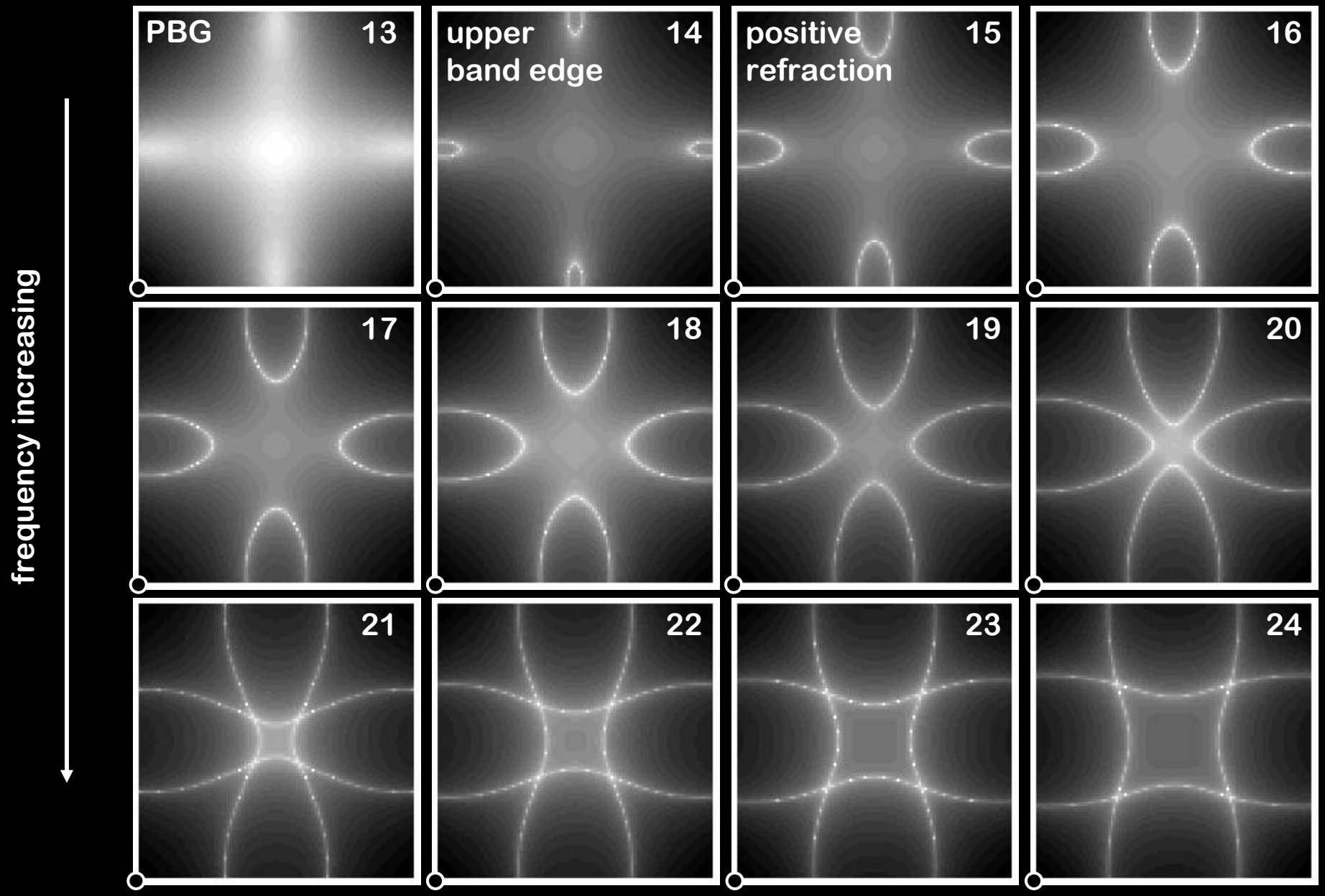
$$\begin{bmatrix} q & k_o^2 M/2 & 0 & k_o^2 M/2 \\ k_o^2 M/2 & q - a_s & k_o^2 M/2 & 0 \\ 0 & k_o^2 M/2 & q - a_s - a_c & k_o^2 M/2 \\ k_o^2 M/2 & 0 & k_o^2 M/2 & q - a_c \end{bmatrix} \begin{pmatrix} V_{00} \\ V_{10} \\ V_{01} \\ V_{11} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

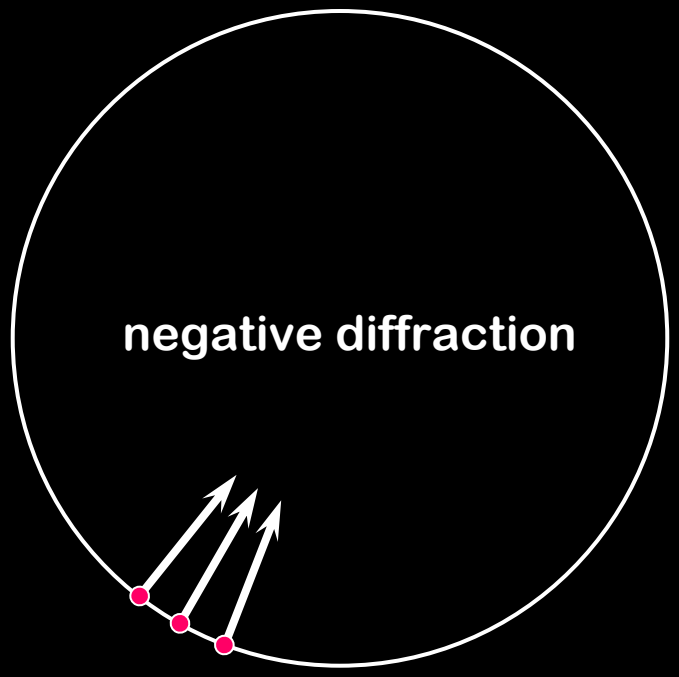
$$a_c = \frac{4\pi}{\Lambda}\left(\frac{\pi}{\Lambda} - (\delta + k_o)\cos\theta\right)$$

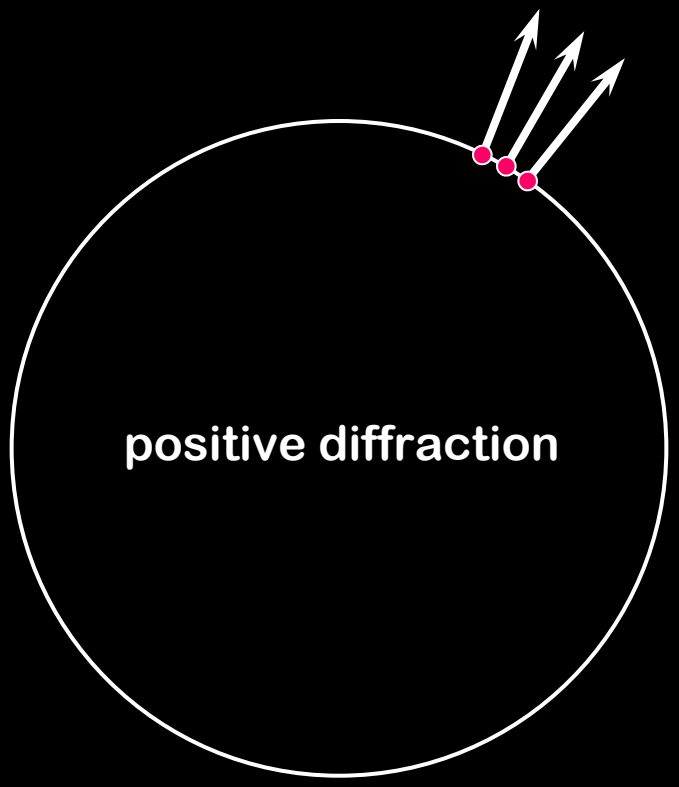
Wavevector diagrams in square crystal



Wavevector diagrams in square crystal







convergent rays:

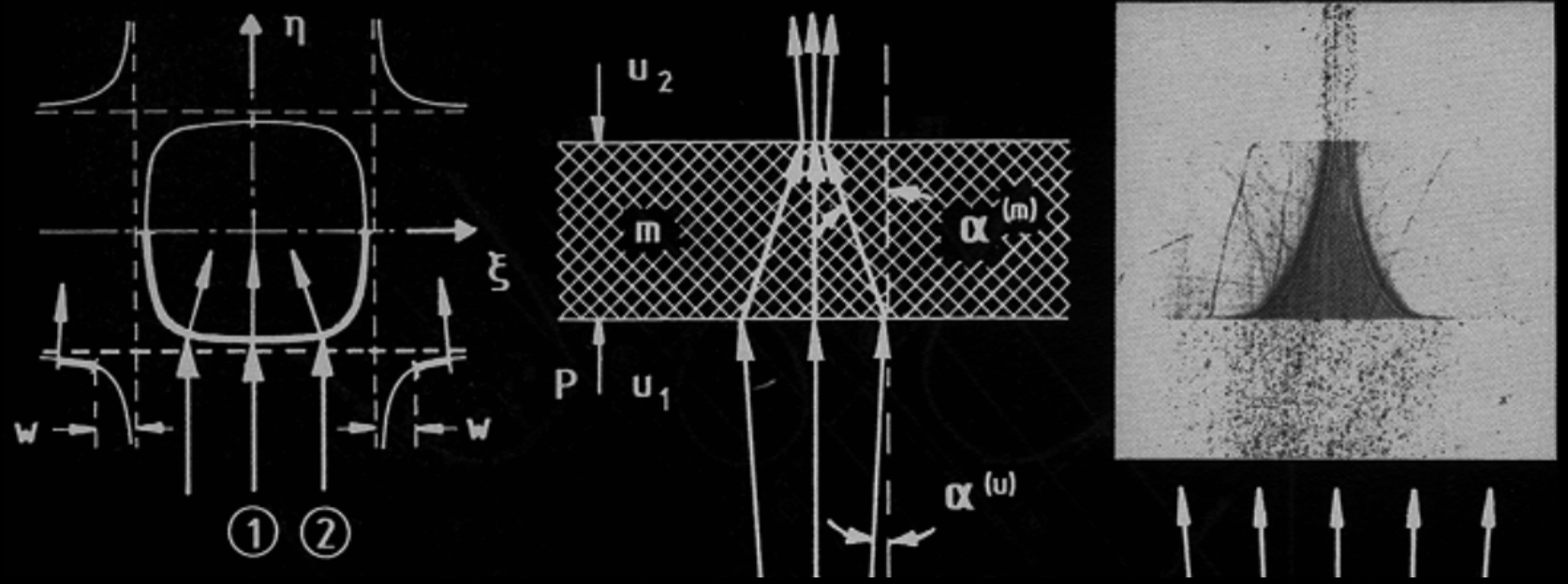
surface “nose” points away from direction of energy flow

divergent rays:

surface “nose” points towards direction of energy flow

Negative diffraction in a square-lattice

Zengerle, J. Mod. Opt. 34 (1589-1617) 1987



frequency ω_1

Negative diffraction in a square-lattice

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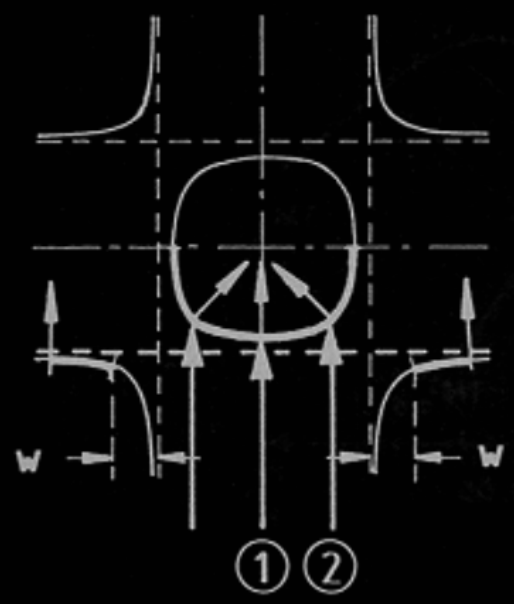
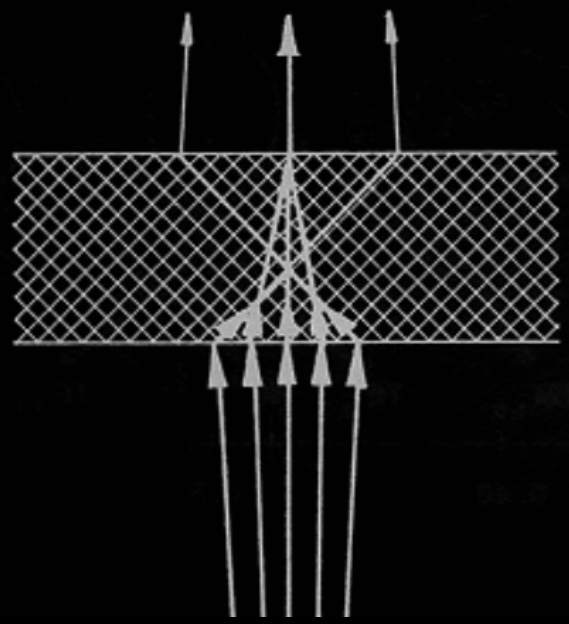
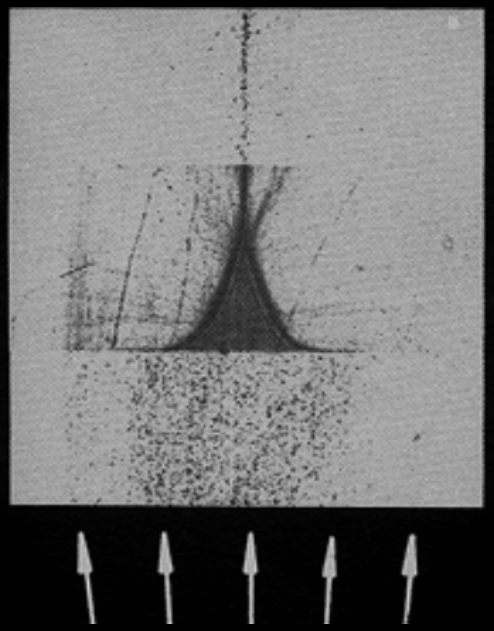


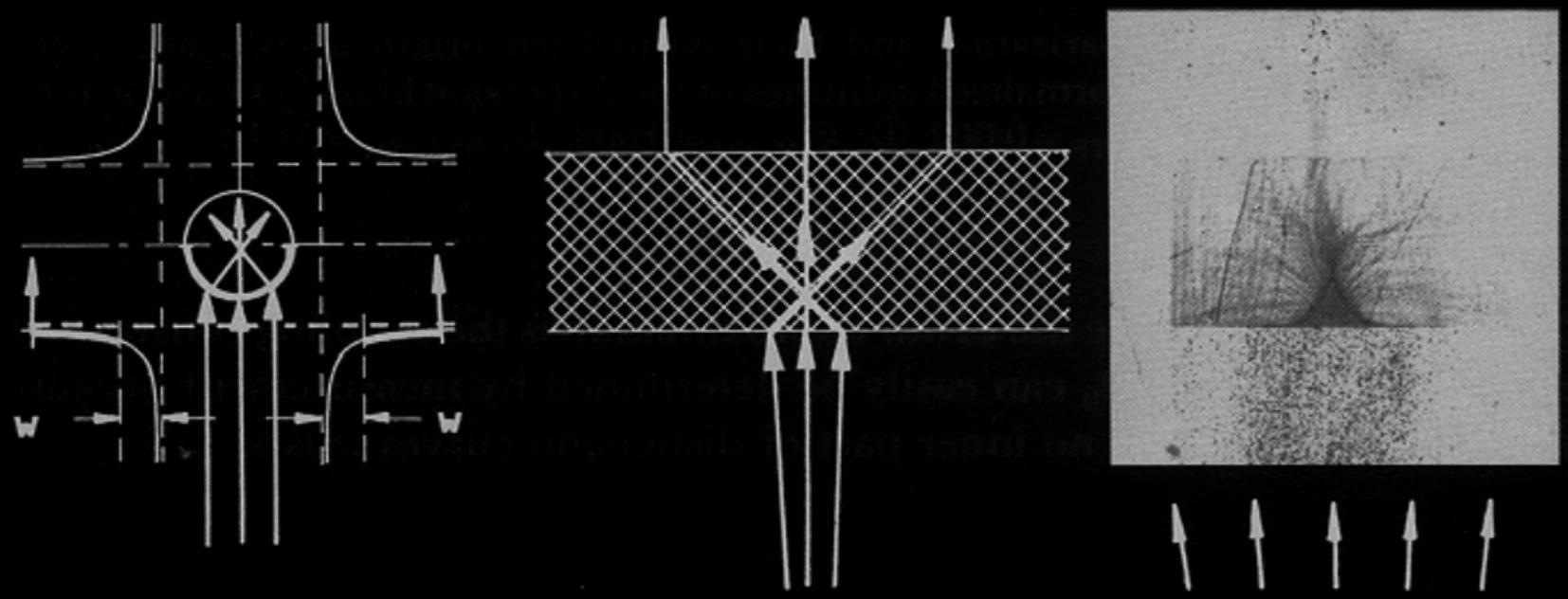




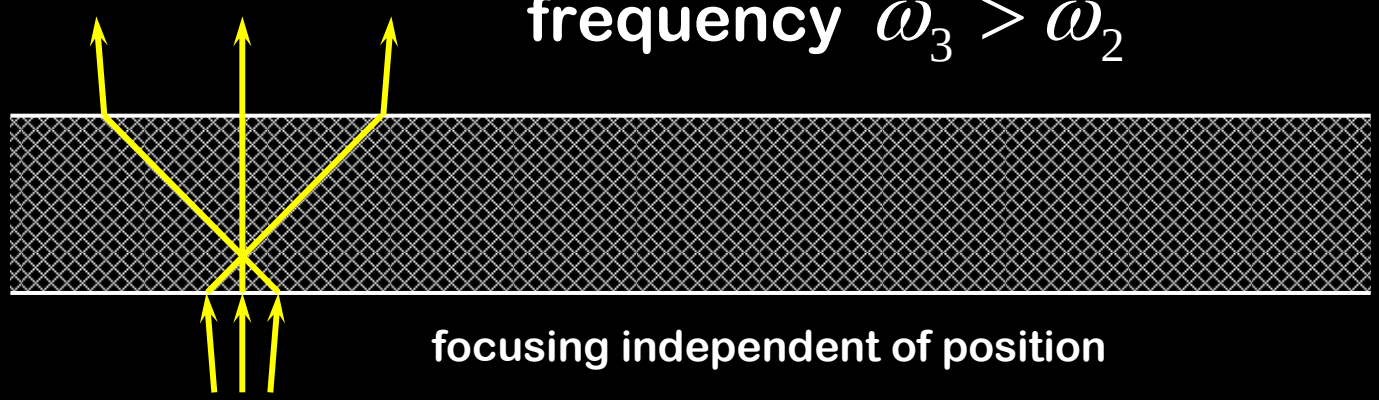
frequency $\omega_2 > \omega_1$

Negative diffraction in a square-lattice

Zengerle, J. Mod. Opt. 34 (1589-1617) 1987



frequency $\omega_3 > \omega_2$



focusing independent of position

Reinhard Ulrich

- first experiments on metal-wire "meta-materials" – early 1970s
- first experiments on multiply-periodic planar waveguides ("photonic crystals") – from 1975

Topics

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Conclusions

- refractive index vector plays a crucial role in periodic media – usually it is multi-valued
- negative & positive refraction are common in periodic media – often at the same time
- negative diffraction is a common feature of propagation in periodic media
- many negative effects were first reported in the early 1980s in planar periodic waveguides

- since about 1990, periodic optical media have become known as “photonic crystals”

http://www.ioip.mpg.de/~russell/PC_website/index.htm