“Be good, be true, be just ...
and remain true to yourself.”

— Motto of Abbe’s life from the
funeral speech of Siegfried Czapski
Last year marked the 100th anniversary of the death of Ernst Abbe, who passed away on January 14, 1906—just a few days short of his 65th birthday. Abbe is well known for his seminal contributions to scientific microscope construction, which include his diffraction theory for image formation, and the formulation of the sine condition and Abbe number.

He was also a noted entrepreneur, astronomer and social reformer.

Ernst Abbe was born in Eisenach, Germany, on January 23, 1840. He spent his childhood in poverty—which perhaps explains why he was committed to social welfare for the rest of his life. After graduating from secondary school, he began his studies at the University of Jena. There, Abbe attended lectures in mathematics and physics.

Early on, his professors noted his acumen in science. Abbe won a scientific competition in his third semester and, later, he was awarded a scholarship by city officials at Eisenach. After completing two years at Jena, Abbe took up three additional years of studies at the University of Göttingen, where he attended lectures in mathematics given by Bernhard Reimann, as well as lectures in meteorology, optics and astronomy.

At Göttingen, Abbe studied the theory and practice of precision measurements (i.e., the assessment of extremely weak electric currents and magnetic fields) as well as the design and construction of measuring instruments and the theory of experimental error.

Abbe remained in Göttingen for his doctoral research under the supervision of two professors: Wilhelm Eduard Weber and Karl Snell. His doctoral thesis focused on experimental substantiation of the theorem of the mechanical equivalence between heat and mechanical energy. He received his doctoral degree in 1861. One year later, he submitted research on the laws of the distribution of errors in an observation series to the philosophy faculty at the University of Jena for his Habilitation paper (a German post-doctoral requirement for teaching as a professor at a university).

Over the next 10 years, Abbe published several scientific papers and was subsequently appointed as a lecturer of astronomy in 1877 at the University of Jena. In addition, Abbe was made the director of the observatory at the University of Jena (1877-1900).

In 1866, Abbe met Carl Zeiss, who proposed that Abbe establish a scientific foundation for the manufacture of optical microscopes. Zeiss had begun his production of single-lens...
dissecting microscopes in Jena in 1847 after Matthias Schleiden—a German botanist and co-founder of the cell theory—suggested that he get into that line of business.

Using lenses purchased from other manufacturers, Zeiss had fabricated in 1857 the first compound microscope consisting of a microscope objective and an ocular. At that time, lenses were ground and combinations were tested for their optical performance; there was no scientific basis for the design of high quality lenses. The historical method of lens grinding, testing and selection was about to change.

The 26-year-old Abbe started work for Carl Zeiss as an independent contractor. Over the next few years, Abbe developed several precision optical instruments to measure the shape and optical constants of lenses. Abbe's locometer measured the focal lengths of lens elements, lens combinations or a complete optical system. The Abbe refractometer measured the refractive index of glasses or liquids based on the angle of total reflection. The Abbe spectrometer measured the refractive index and dispersion of various glasses. In 1870, Abbe developed his apertometer, which measured the numerical aperture of microscope objectives.

Microscope design followed in the footsteps of the Keplerian telescope, and multi-lens oculars were rapidly adapted from those of telescopes from Ramsden and Huygens. Another key example is the problem of chromatic aberration, which was solved by Isaac Newton in his reflecting telescopes that incorporated a concave mirror in place of a lens. Soon afterward came the development of reflecting microscopes.

Abbe eventually became a co-owner of the firms Zeiss and Schott. He did not accept any salary for his work as the director of the observatory and as a university teacher so that less fortunate staff members could benefit from the funds. A major breakthrough for Abbe as well as the Zeiss factory was the 1870 formulation of the Abbe sine condition, which determines the design of a spherically corrected lens that is free from coma (an optical aberration).

By 1871, Abbe had completed his calculations for the design of microscope objectives, and, in 1872, these calculations permitted the Zeiss factory to manufacture water immersion microscope objectives of much higher quality than was previously possible. Simultaneously, Abbe worked on his theoretical calculations. He designed and performed practical demonstrations that verified as well as made his theory of microscope image formations based on diffraction theory clear to a wide audience.

The important result for microscope objective design and production, was that—for the first time—the theoretical optical basis for microscope design had matched the theoretical foundations that were used in telescope lens design. That was a major milestone because the production of microscope objectives and other microscope optical components had previously been a trial-and-error process.

The next step was to improve the glass that was used in the manufacture of lenses—which required the manufacture of new types of optical glasses with the correct dispersion (change of refractive index with frequency of the light). With the acquisition of the Schott glass works, this became a reality. By 1884, Abbe, Otto School and Carl and Roderich Zeiss (the son of Carl Zeiss) joined forces in Jena and formed the company that today is Schott.

The success of the Zeiss and Schott factories in the subsequent years was in large part based on the “Fraunhofer method,” which includes: (1) the development and perfection of manufacturing methods; (2) the mathematical formulation of the fundamental theories; and (3) the improvement of the raw materials (in this case glass). Fraunhofer produced a series of telescope objectives from his secret recipes for special glasses; however, when he attempted to construct microscope objectives, he failed to achieve the same achromatic qualities that characterized his telescope objectives.

It was Fraunhofer who worked on the problem of minimizing chromatic aberrations in optical systems through the development of new types of glasses. Fraunhofer developed a method to measure the refractive indices of glasses and used the dark lines of the solar spectrum for calibrating the lines. Later, he developed a diffraction grating to directly measure the frequencies of the spectral lines.

Abbe is also respected for his pioneering work to improve workplace conditions at the Zeiss factories. Beginning in 1887, he helped to implement several innovative reforms that employees today take for granted: a pension fund for staff and their dependents, an eight-hour work day, paid vacations, a company
health insurance plan, sick pay and company regulations to prohibit discrimination due to ethnicity, religion or political affiliation.

In 1889, Abbe set up the Carl Zeiss Foundation as the sole owner of the Zeiss and Schott factories. The Foundation’s rules stipulated that the university and the city of Jena should benefit from the profits. The city responded by awarding Abbe an honorary doctoral degree from Jena in 1896. At Fürstengraben, the University of Jena erected a memorial to Abbe consisting of a stone globe, on which the Abbe resolution formula for oblique illumination is carved.

From about 1890, the Carl Zeiss Foundation began to manufacture a wide range of optical instruments in addition to their line of optical microscopes: optical measuring instruments, camera lenses, binoculars, astronomical instruments and photometric instruments. Abbe laid the scientific foundation for the design, manufacture and testing of these optical systems.

Beginning in 1887, Abbe helped to implement several innovative reforms that employees today take for granted: a pension fund for staff and their dependents, an eight-hour workday, paid vacations, a company health insurance plan, sick pay and company regulations to prohibit discrimination due to ethnicity, religion or political affiliation.

It is important to also cite the seminal contributions of August Köhler, who in 1893 developed a new system of illumination for the optical microscope. This illumination method—which came to be known as Köhler illumination—permitted the microscopist to use the full resolving power of Abbe’s microscope objectives.

**Abbe’s sine condition**

Abbe’s early progress in developing microscopes at Zeiss was hindered by the prevailing—and false—belief that the problems of chromatic aberrations in high-magnification microscopes could be solved using the same approaches that worked for telescope design. In telescopes, the optical aberrations off-axis are unimportant because the celestial bodies they were designed to examine are self-luminous objects at infinite distance with a negligible angular aperture. For microscopes, however, the conditions differ and the off-axis aberrations are large.

Abbe addressed this issue by designing lenses with minimum off-axis aberrations with his theoretical conception of the Abbe sine condition. The light from two point stars is incoherent, whereas, in a microscope, the light from the object is coherent or partially coherent and thus can interfere.

At first, Abbe calculated the image of a point on the optical axis in the image space from the corresponding point on axis in object space. Then, he realized that there must be a constant magnification for all points on the surface of an object that slightly extends from the optical axis. Finally, he deduced the mathematical relationship between the bundle of rays from an object and the bundle of rays on the image side of a lens.

For conjugate points (i.e., the same points in the object and in the image), Abbe found that the ratio of the sines of the two angles of the bundle of rays in object space and in image space (on the two sides of a lens) must be a constant over the full aperture of the optical system. This consideration is called the Abbe sine condition. It is important in optical design because it provides an optical system that is corrected for spherical aberration and that can image points on a small surface element perpendicular to the optical axis without coma.

Abbe’s development of his sine condition provided the theory to design aplanic microscope objectives that are free from both spherical aberration and coma. With this condition fulfilled, a lens or an optical system produces sharp images both on and off the optical axis; however, the condition does not hold for Fresnel lenses where the rays are not continuous.

The sine condition required the magnification of the marginal and paraxial rays to be equal. It permitted Abbe to calculate if changes in the design of a lens would improve or worsen the image quality of an optical system. These calculations resulted in the better, more advanced microscopes that became available at the Zeiss factory in 1871.

The microscopes shown above were produced by Zeiss after Abbe had developed his theory of image formation in 1873. The model on the facing page, called the labormikroskop ("laboratory microscope"), was made in 1881. The other was the Großes Forschungsmikroskop ("large research microscope"); it was made in 1885. [Images courtesy of Timo Mappes, www.musoptin.com.]
In 1873, Abbe published his landmark paper in a biological journal that explained the role of wavelength and aperture of the microscope objective on microscope resolution—the ability to resolve two closely positioned points on an object as separate points.

**Abbe number**

Abbe also defined a special number called the Abbe number. Also sometimes referred to as the V-number of a transparent material, the Abbe number is a measure of the variation of the refractive index with frequency (also called the dispersion). The Abbe number, V, of a material is defined as: \( V = (n_D - 1)/(n_F - n_C) \), where \( n_D \), \( n_F \), and \( n_C \) are the refractive indices of the material at the Fraunhofer D, F, and C spectral lines (589.2 nm, 486.1 nm and 656.3 nm). Materials that have a low dispersion have large values. Abbe numbers are used to classify various types of glass; low dispersion glass has a high Abbe number. However, the number is only useful in measuring the dispersion of visible light. An Abbe diagram is a plot of the Abbe number V of a glass versus the refractive index of the glass \( n_D \). It is used to define different glasses in the Schott catalogue.

**Abbe’s theory of image formation**

In the 1840s, the Italian microscopist Giovan Battista Amici invented the water immersion microscope objective. Since the microscope’s apertures were still small, the image quality was not optimal. One problem was that only a thin bundle of rays illuminated the objective; thus, the illuminating light did not fill the back aperture of the microscope objective.

At first, Abbe produced microscope objectives with very small angular apertures. While these objectives had improved spherical and chromatic corrections (less of these two aberrations), they were less bright and had a lower resolution than similar microscope objectives made with large angular apertures. Abbe realized that he could explain this result by the diffraction of the illumination light by the objects in the specimen whose dimensions are similar to the wavelength of the illumination light. The angles of the diffracted light then completely fill the numerical aperture of the objective.

In 1873, Abbe published his landmark paper in a biological journal that explained the role of wavelength and aperture of the microscope objective on microscope resolution—the ability to resolve two closely positioned points on an object as separate points.

Abbe’s article comprised 55 pages without any equations or diagrams. Abbe first considered the diffraction of light by the object and then the effect of the aperture. In 1896, Lord Rayleigh published a paper in which he first considered the aperture and then the object, but he reached similar conclusions to Abbe’s. Rayleigh used Lagrange’s theorem and Fourier analysis in order to calculate the diffraction pattern of apertures with various shapes, as well as the diffraction pattern from gratings.

Abbe described several experiments using a sinusoidal grating as an object. He observed the diffraction pattern of the grating with the ocular removed and with various stops placed in the microscope to alter the number of diffraction orders that produced the diffraction pattern. Each of the diffracted beams was called a “spectra” due to their appearance with white light illumination. It was the combination of all the diffraction orders that formed the image.

In other words, with more diffraction orders from the object entering the microscope objective, the observer could resolve more detail. Abbe also noted that oblique illumination increased the resolution of the microscope; this was because a higher order of diffraction entered the objective when the central illumination beam was shifted to one edge of the objective by tilting the illumination with respect to the optical axis. In the back focal plane of the objective, each Airy disk is a source that forms a spherical wave; the spherical waves interfere in the image plane to form the image of the object.

In 1876, Abbe traveled to London with his gratings, apertures and microscope in order to demonstrate his diffraction theory in front of the members of the Royal Microscopical Society.
Society. The theme that Abbe promoted with his experiments was that microscopic imaging of structures whose dimensions are similar to the wavelength of light cannot be explained on the basis of geometrical optics; rather, the phenomena required diffraction and interference effects.

Abbe was able to set the limit of the resolution (later called the Abbe limit of resolution) of the optical microscope from his requirement that both the central (0th order of diffraction) and at least one of the diffraction order maxima must enter the objective to achieve maximum resolution. The nondiffracted 0th order rays and the qth order rays are separated in the back focal plane (diffraction plane) and combined in the image plane. Abbe calculated this for an object that consisted of a periodic structure (lines) for a non-immersion microscope objective, and a circular aperture (the microscope objective lenses) using direct illumination as: 

\[d = \frac{\lambda}{(2n \sin \alpha)}\]

for oblique illumination, where \(d\) is the smallest separation that can be resolved, \(\lambda\) is the wavelength of the illumination light in vacuum, and \(\alpha\) is one-half of the angular aperture of the microscope objective.

Abbe is also credited with the formulation of the term “numerical aperture.” In microscopy, the numerical aperture, or NA, of a microscope objective is given as: 

\[NA = n \sin \theta\]

where \(n\) is the refractive index of the medium between the object and the microscope objective, and \(\theta\) is one-half of the angle of the cone of light that can enter the objective. The angular aperture of the lens is twice the angle \(\theta\).

When other microscopists objected to Abbe’s theory claiming that they could observe smaller details, Abbe replied that they were observing false detail—artifacts that were not actually part of the object. The presence of the refractive index in the denominator offers another way to increase the resolution of the microscope (reduce the separation distance that could be resolved)—the concept of oil immersion. If we use oil (\(n=1.5\)), the resolution will be increased as compared to air (\(n=1\)).

In 1874, Helmholtz published a paper in a physics journal in which he calculated the maximum resolution of an optical microscope. Helmholtz used ray tracing methods that were long used in telescope design and concluded that the smallest separation of two distinct points in the object that could be resolved was equal to one-half of the wavelength of the illumination light (for light of 500 nm, the resolution is 250 nm). After writing his paper, Helmholtz came across Abbe’s prior article on the same topic. He attached a note to his paper that acknowledged the priority of Abbe.

Within a decade, others published articles explaining the Abbe diffraction of image formation in a microscope and included illustrations of the patterns described in the earlier Abbe experiments.

In 1906, Porter described the same theory and Abbe’s experiments in only 12 pages because he invoked Fourier’s theorem. Thus, he demonstrated the power of Fourier optics. These experiments were repeated and published in 1906 by Porter. When the ocular is removed, the spectral bands of the grating (object) are observed in the back focal plane of the objective lens. By placing different obstructions in the back focal plane (diffraction plane), the image could be altered (spatial filtering).

We can summarize the Abbe theory as follows: In an optical system, there are two causes of aberration; the first is spherical and chromatic aberration, which have their origins in properties of lenses (i.e., refractive indices, curvatures, etc.), and the second is the effect of light diffraction. It is the latter phenomena that is the foundation of the Abbe theory of image formation in the microscope.

[ Barry R. Masters (bmasters@mit.edu) is with the Biological Engineering Division at the Massachusetts Institute of Technology in Cambridge, Mass. ]