

Finding Slope on Semi Logarithmic Graph

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We found experimentally unknown value from slope of graph while relation between variables is linear like $\mathbf{F} = \mathbf{k} \cdot \mathbf{x}$ or $\mathbf{V} = \mathbf{I} \cdot \mathbf{R}$ in previous experiments. However, we have exponential relation between current (i) and time (t) as shown in **Equation 1** in 'Resistance-Capacitance Circuits' experiment. Therefore if we draw logarithmic graph, we can find unknown value by slope again since we convert relation to linear one in this way.

$$i = i_0 \exp\left(\frac{-t}{RC}\right) \quad (1)$$

Unfortunately, the slope does not give unknown directly. If we look at the lab manual, a hint is given actually about the subject



Hint: Note that, if the current i is plotted against the time on a semi-logarithmic scale, the i(t) curve is a straight line with negative slope $m = -\log_{10} e (RC)^{-1}$ and the intercept at the current axis is $\log_{10} i_0$ that is;

$$\log_{10} i = \log_{10} i_0 - \frac{\log_{10} e}{RC} t, \quad \text{where } \log_{10} e \approx 0.4343$$

We can derive what RC value is equal to by getting help from hint. However, it is better to start with definition of slope in linear relation and it is

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad (2)$$

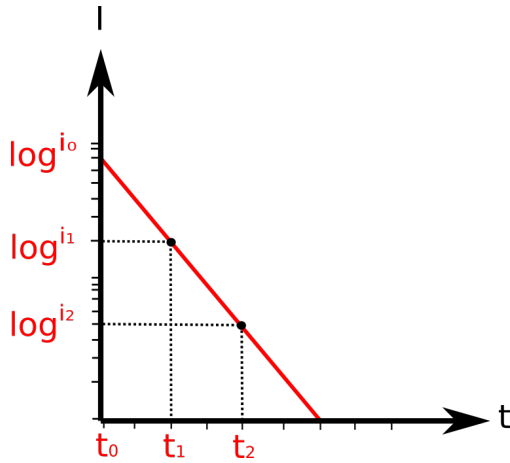


Figure 1: Representation of I-t Graph (Not Real)

In semilog case, if we choose y_2, y_1, x_2 and x_1 as $\log^{i_2}, \log^{i_1}, t_2$ and t_1 , slope becomes

$$m = \frac{\log(i_2) - \log(i_1)}{t_2 - t_1} \quad (3)$$

In this step, we have to find what m is equal to in **Equation 1** since it consists of some constants and it might affect due to logarithmic conversion. Therefore, we begin with **Equation 1** by writing it in logarithm form at base 10;

$$\log(i) = \log(i_0 e^{(-t/RC)}) = \frac{-t}{RC} \log(e) + \log(i_0) \quad (4)$$

We obtain second formula given in the hint. If we write it in the differential form, the equation looks like

$$m = \frac{\log(i_2) - \log(i_1)}{t_2 - t_1} = -\frac{\log(e)}{RC} \quad (5)$$

$\log(i_0)$ is canceled since it is constant so it gives zero in in the differential form. Then, we can find experimental RC value as

$$RC = -\frac{\log(e)}{m} = -\frac{0.4343}{m} \quad (6)$$

Warning: In lab manual, plot is not I-t but reverse. It is not a problem if you rotate the page and draw I-t graph on landscape orientation so that current(I) becomes on y-axis and also on semilog side.