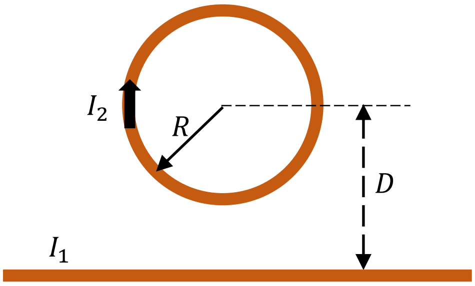
PHYS 102 – General Physics II Midterm Exam 2 Solutions

Duration: 90 minutes | Saturday, 14 December 2019; 14:00

1. A circular loop has radius R and carries current I_2 in a clockwise direction. The center of the loop is a distance D above an infinitely long, straight wire carrying a current I_1.



(a) (8 Pts.) What is the direction of the current I_1 in the straight wire (to the right or to the left) if the magnetic field at the center of the loop is zero? Explain.

(b) (9 Pts.) If $I_2 = I$, what is the magnitude of the magnetic field created by the loop at the center of the loop?

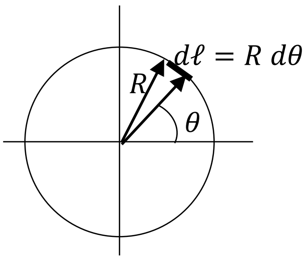
(c) (9 Pts.) If $I_1 = I_2 = I$ and the direction of I_1 is to the left, what is the magnitude of the total magnetic field at the center of the loop?

(d) (9 Pts.) If $I_1 = I_2 = I$ and the direction of I_1 is to the left, is the magnetic force between the loop and the wire attractive or repulsive? Explain.

Solution:

(a) Magnetic field $\vec{\boldsymbol{B}}_2$ created by I_2 is perpendicular and into the plane of the page. So $\vec{\boldsymbol{B}}_1$ created by I_1 must be out of the page. This means I_1 must be to the right.

(b)



$$d\vec{\boldsymbol{B}}_2 = \frac{\mu_0 I}{4\pi}\frac{\boldsymbol{d}\vec{\boldsymbol{\ell}} \times \hat{\boldsymbol{r}}}{r^2} \quad \rightarrow \quad \left|d\vec{\boldsymbol{B}}_2\right| = \frac{\mu_0 I}{4\pi}\frac{R\,d\theta}{R^2}$$

$$\left|\vec{\boldsymbol{B}}_2\right| = \frac{\mu_0 I}{4\pi R}\int_0^{2\pi} d\theta \quad \rightarrow \quad B_2 = \frac{\mu_0 I}{2R}$$

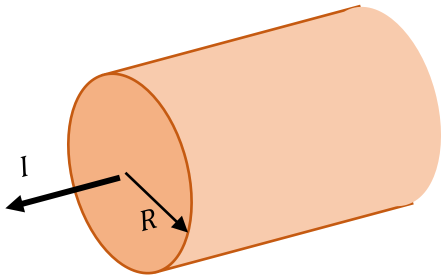
(c) Using Ampère's law around a circle C of radius D enclosing the current I_1, we have

$$\oint_C \vec{\boldsymbol{B}}_1 \cdot \boldsymbol{d}\vec{\boldsymbol{\ell}} = \mu_0 I_1 \quad \rightarrow \quad B_1 = \frac{\mu_0 I_1}{2\pi D}$$

$$\left|\vec{\boldsymbol{B}}_{total}\right| = \frac{\mu_0 I}{2}\left|\frac{1}{R} + \frac{1}{\pi D}\right|$$

(d) If the direction of I_1 is to the left, the magnetic field created by I_1 is perpendicular and into the plane of the page. Net force is only in the vertical direction because of the symmetry of the loop. At the bottom point where the two currents are closest currents are parallel and the force on the bottom section is attractive. Since the top portion where the currents are antiparallel is farther away, the net force on the loop is attractive.

2. A cylindrical conductor with radius R carries a current I. The current is uniformly distributed over the cross-sectional area of the conductor.

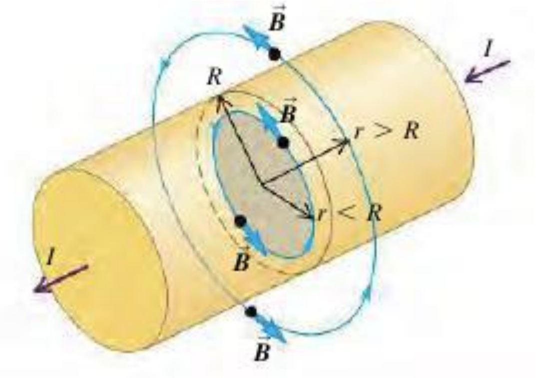


(a) (10 Pts.) What is the magnitude of the current density $\vec{J}$?

(b) (10 Pts.) Find the magnitude of the magnetic field as a function of the distance r from the conductor axis for points inside $(r < R)$ the conductor.

(c) (10 Pts.) Find the magnitude of the magnetic field as a function of the distance r from the conductor axis for points outside $(r > R)$ the conductor.

Solution: (Example 28.8 of the textbook)



(a) Current density inside the wire is

$$|\vec{\mathbf{J}}| = \frac{I}{\pi R^2}$$

(b) Using Ampère's law around a circle C of radius r where $0 < r < R$ centered on the symmetry axis of the wire, we have

$$\oint_C \vec{\boldsymbol{B}} \cdot \boldsymbol{d}\vec{\ell} = \mu_0 I_{enc} \quad \rightarrow \quad B = \frac{\mu_0 I_{enc}}{2\pi r}$$

Since

$$I_{enc} = J\,\pi r^2 = \frac{I r^2}{R^2}$$

$$B = \frac{\mu_0 I\, r}{2\pi R^2}, \qquad 0 < r < R$$

(c) For a circle C of radius r where $0 < r < R$ centered on the symmetry axis of the wire, we have $I_{enc} = I$. Hence

$$B = \frac{\mu_0 I}{2\pi r}, \qquad R < r < \infty$$

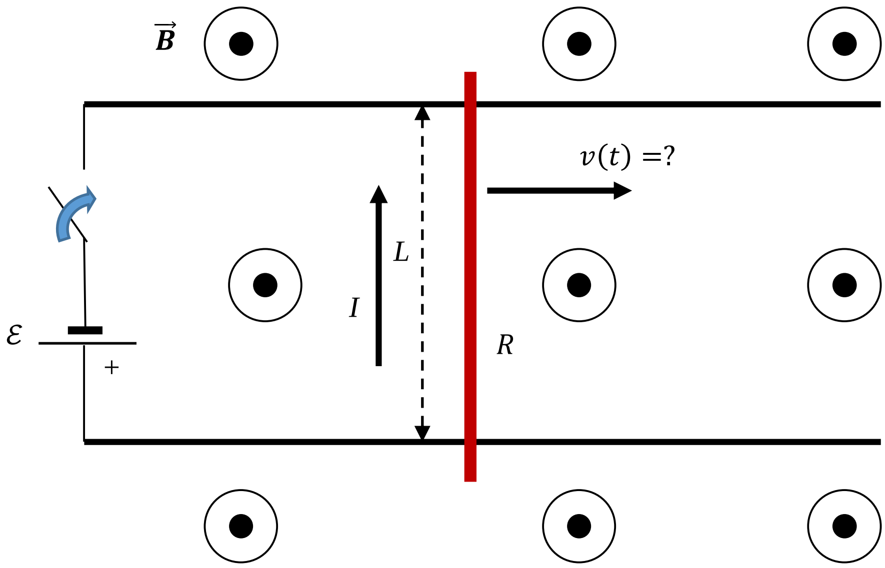
3. A rod of mass M and length L can slide without friction on two parallel rails as shown in the figure. There is a uniform magnetic field $\vec{B}$ directed out of the plane. The rails have negligible resistance but the rod has resistance R. Initially the rod is at rest, but at time $t = 0$ the switch is closed connecting the rod to a battery of $\mathcal{E}$ volts.

(a) (8 Pts.) Find the direction and magnitude of the force acting on the rod at time $t = 0$.

(b) (9 Pts.) Find the current in the system if the rod is moving right with speed v.

(c) (9 Pts.) What is the maximum (terminal) velocity that the rod will reach?

(d) (9 Pts.) Find the velocity of the rod as a function of time.



Solution:

(a)

$$I = \frac{\mathcal{E}}{R}, \qquad F = BIL \quad \rightarrow \quad F = \frac{B\mathcal{E}L}{R}$$

Direction of the force is to the right.

(b) Once the bar starts moving with speed v to the right, there will be in induced emf $\mathcal{E}_{ind} = BLv$ on it in the opposite direction (Lenz's law). Hence the current will be

$$I = \frac{\mathcal{E} - BLv}{R}$$

(c) Terminal velocity v_T is reached when the current in the rod is zero, hence there is no force acting on the rod.

$I = 0$ means

$$v_T = \frac{\mathcal{E}}{BL}$$

(d) At any time t, the force on the bar is

$$F(t) = BLI(t) = \frac{BL}{R}[\mathcal{E} - BLv(t)]$$

Since $F = Ma = M\frac{dv}{dt}$, we have

$$M\frac{dv}{dt} = \frac{BL}{R}[\mathcal{E} - BLv(t)] \quad \rightarrow \quad \frac{dv}{\mathcal{E} - BLv} = \frac{BL}{MR}dt \quad \rightarrow \int_0^v \frac{dv'}{\mathcal{E} - BLv'} = \frac{BL}{MR}\int_0^t dt'$$

Integrating both sides, we get

$$v(t) = \frac{\mathcal{E}}{BL}\left(1 - e^{-\frac{B^2L^2}{MR}t}\right)$$