PHYS 102 – General Physics II Midterm Exam 2 Solutions

01 December 2018

1. Light bulb of an automobile dissipates 60 Watts when connected to a 12 Volt battery.

(*a*) (7 Pts.) What is the resistance of the light bulb?

(*b*) (6 Pts.) Calculate the current through the light bulb when it is in operation.

(*c*) (6 Pts.) If the battery is rated as 60 Ampère-Hours what is the total charge the battery can supply before it runs out?

(*d*) (6 Pts.) How many hours will it take the light bulb to completely drain the battery?

Solution:

(*a*) $P = \frac{V^2}{R} \rightarrow R = \frac{V^2}{P} = \frac{(12\text{ V})^2}{60\text{ W}} = 2.4\,\Omega$.

(*b*) $I = \frac{P}{V} = \frac{60\text{ W}}{12\text{ V}} = 5\text{ A}$.

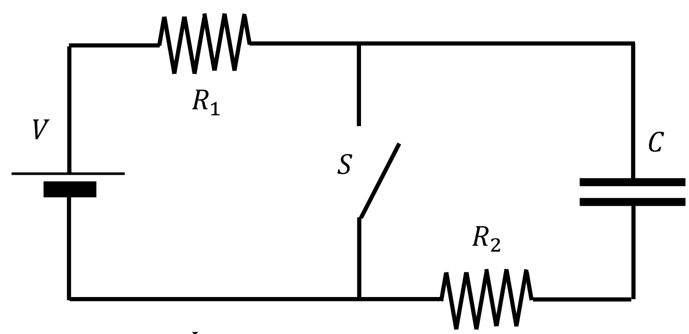
(*c*) $Q = 60\text{ Amp. Hrs} = (60\text{ A})(3600\text{ s}) = 216000\text{ C}$.

(*d*) $T = \frac{60\text{ A.hr}}{5\text{ A}} = 12\text{ hours}$.

2. In the circuit given the switch S has been left open for a very long time.

(*a*) (5 Pts.) What is the charge on the capacitor?

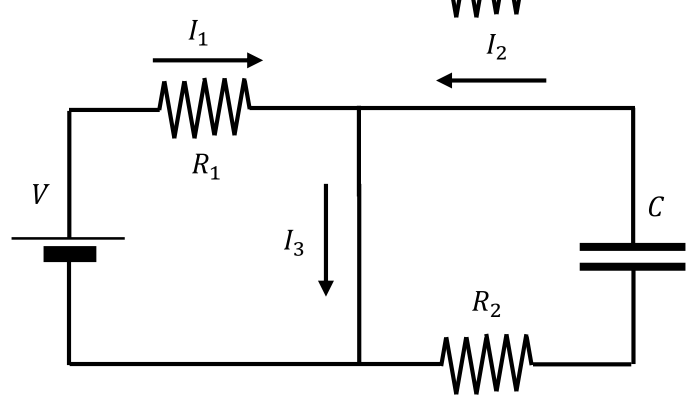
(*b*) (20 Pts.) Find the current through the switch *S* as a function of time after it is closed at time $t = 0$.



Solution:

(*a*) $Q_0 = VC$.

(*b*) Applying Kirchhoff's rules to the circuit, we write



$I_3 = I_1 + I_2$, $V - I_1 R_1 = 0$, $V_C - I_2 R_2 = 0$

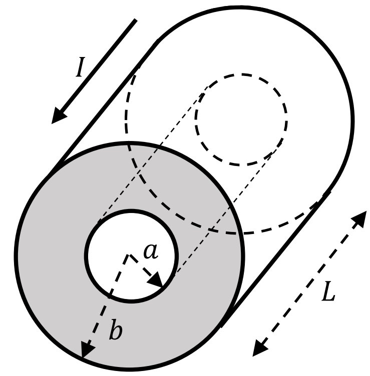
Since the capacitor is being discharged, we have

$I_2 = -\frac{dQ}{dt}$ and $V_C = \frac{Q}{C}$. Hence

$$\frac{Q}{C} + R_2 \frac{dQ}{dt} = 0 \rightarrow \frac{dQ}{Q} = -\frac{dt}{R_2 C} \rightarrow Q(t) = Q_0\, e^{-t/(R_2 C)}. \text{ So } I_2 = -\frac{dQ}{dt} = \frac{Q_0}{R_2 C} e^{-t/(R_2 C)} = \frac{V}{R_2} e^{-t/(R_2 C)}.$$

$$I_3 = I_1 + I_2 \rightarrow = I_3 = \frac{V}{R_1} + \frac{V}{R_2} e^{-t/(R_2 C)}.$$

3. An infinite hallow wire is used to carry current along its length. The inside of the wire ($r < a$) is empty, while the region between the inner and outer radii ($a < r < b$) is filled with a material of resistivity ρ.



(a) (5 Pts.) What is the resistance R of a section of the wire that has length L?

If the wire is carrying a total current of I, which is uniformly distributed,

(b) (5 Pts.) What is the magnitude J of the current density within the wire?
(c) (15 Pts.) What is the magnitude of the magnetic field for $a < r < b$?

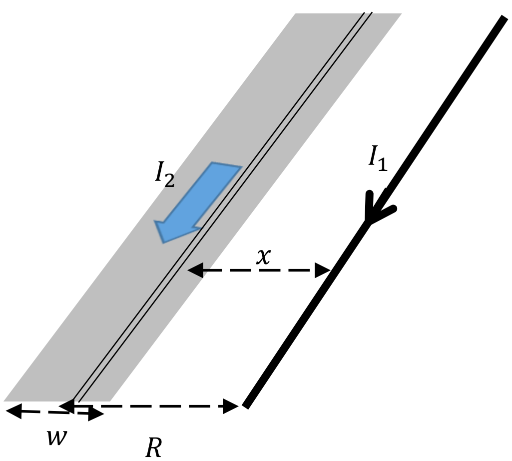
Solution:

(a) $R = \rho \dfrac{L}{A} \rightarrow R = \rho \dfrac{L}{\pi\left(b^2 - a^2\right)}$ (b) $J = \dfrac{I}{A} \rightarrow J = \dfrac{I}{\pi\left(b^2 - a^2\right)}$

(c) Consider a circular path of radius r, $a < r < b$, centered at the symmetry axis. Applying Ampère's law, we have

$$\oint_C \vec{\mathbf{B}} \cdot d\vec{\ell} = \mu_0 I_{enc} \rightarrow B(2\pi r) = \mu_0 J \pi (r^2 - a^2) = \frac{\mu_0 I (r^2 - a^2)}{\left(b^2 - a^2\right)}. \text{ Hence } B(r) = \frac{\mu_0 I}{2\pi r} \frac{(r^2 - a^2)}{\left(b^2 - a^2\right)}.$$

4. (25 Pts.) An infinite, thin strip of width w carries a current of I_2 distributed uniformly across its surface. In the same plane with the strip and at a distance R from its center is an infinite wire which carries a current I_1.



Find the force per unit length between the strip and the wire. Indicate both the direction and the magnitude of the force on the wire.

Solution: If we divide the strip into infinitesimal slices of thickness dx parallel to I_1, current through each strip will be $dI = \dfrac{I_2}{w} dx$. Force per unit length between the infinitesimal piece at a distance x and I_2 is

$$dF = \frac{\mu_0 I_1 \, dI}{2\pi x} = \frac{\mu_0 I_1 I_2}{2\pi w} \frac{dx}{x},$$ and is attractive because the currents are parallel. Integrating this expression, we find

$$F = \frac{\mu_0 I_1 I_2}{2\pi w} \int_{R-\frac{w}{2}}^{R+\frac{w}{2}} \frac{dx}{x} \text{ which gives } F = \frac{\mu_0 I_1 I_2}{2\pi w} \ln\left(\frac{R + \frac{w}{2}}{R - \frac{w}{2}}\right).$$