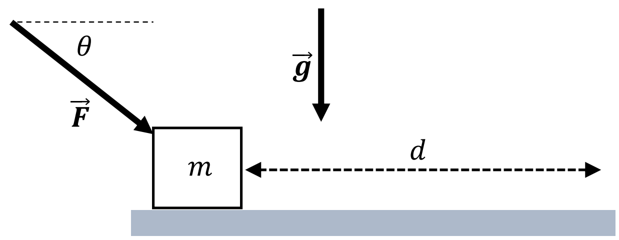
PHYS 101 – General Physics I Midterm Exam 2

Duration: 120 minutes Saturday, 15 April 2017, 14:00

1. A force of magnitude $F = 25$ N acts on a block of mass $m = 0.5$ kg on a horizontal table, as shown in the figure. Under the action of this force, the block, starting from rest, accelerates to $v = 10$ m/s, moving a horizontal distance of $d = 10$ m. (Take $\cos\theta = 3/5$)



(*a*) (6 Pts.) What is the work done on the block by the force $\vec{\mathbf{F}}$?

(*b*) (6 Pts.) What is the work done on the block by the gravitational force?

(*c*) (6 Pts.) What is the coefficient of kinetic friction between the mass and the horizontal surface?

(*d*) (7 Pts.) What is the instantaneous power delivered to the block by the force $\vec{\mathbf{F}}$ as a function of time?

Solution:

(*a*) $W_F = Fd\cos\theta = (25\,\text{N})(10\,\text{m})(3/5) = 150\,\text{J}$.

(*b*) $W_g = mgd\cos(\pi/2) = 0$.

(*c*) $\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}\left(\frac{1}{2}\text{kg}\right)(10\,\text{m/s})^2 - (0) = 25\,\text{J}$.

According to the work-energy principle $W_F + W_{fric} = \Delta K \rightarrow W_{fric} = \Delta K - W_F = 25\,\text{J} - 150\,\text{J} = -125\,\text{J}$.

Since $W_{fric} = -\mu_k F_N d = -\mu_k(mg + F\sin\theta)d = -\mu_k\left[(5\,\text{N}) + (25\,\text{N})(4/5)\right](10\,\text{m}) = -(250\,\text{J})\mu_k$,

we get $\mu_k = \frac{-125\,\text{J}}{-250\,\text{J}} = 0.5$.

(*d*) $P = \vec{\mathbf{F}}\cdot\vec{\mathbf{v}} = Fv\cos\theta$.

The block accelerates from rest to $v = 10$ m/s traveling a distance $d = 10$ m. Therefore, its acceleration is

$$v_f^2 - v_i^2 = 2ad \quad \rightarrow a = \frac{(10\,\text{m/s})^2}{2(10\,\text{m})} = 5\,\text{m/s}^2.$$

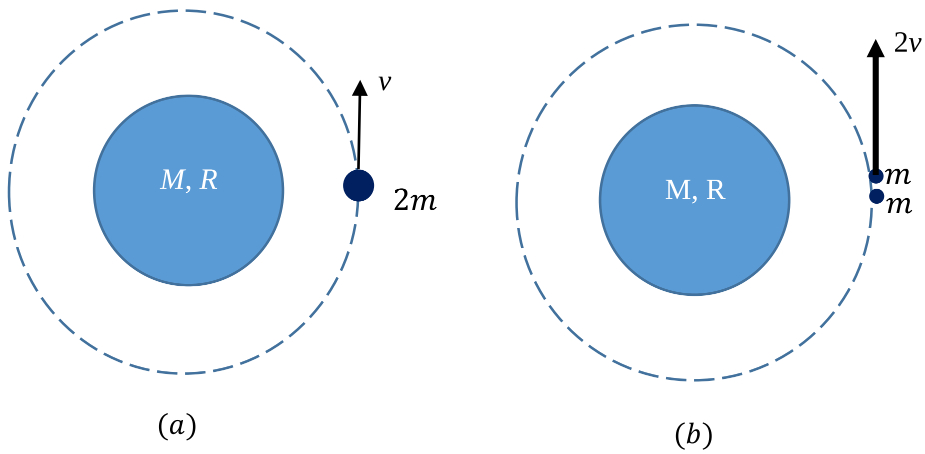
Therefore $v = at \rightarrow v = (5\,\text{m/s}^2)t$, and

$$P = \vec{\mathbf{F}}\cdot\vec{\mathbf{v}} = Fv\cos\theta = (25\,\text{N})(5\,\text{m/s}^2)t(3/5) = 75t\,\text{W}.$$

2. A satellite of mass $2m$ is in a circular orbit with speed v, around a planet of mass M ($m \ll M$) and radius R. The planet has no atmosphere (Figure (*a*)). Answer the following in terms of *M*, *m*, *R*, *G* and *v*.

(*a*) (10 Pts.) What is the radius of the orbit?

(*b*) (15 Pts.) Suddenly an explosion inside the satellite breaks it up into two pieces of equal mass (Figure (*b*)). One of the pieces leave the explosion with speed $2v$ in the direction of velocity before the explosion. Find the speed of the other piece when it collides with the planet.



Solution:

(*a*) Since $F_G = G\frac{M\,(2m)}{r^2} = (2m)\frac{v^2}{r}$, the radius of the orbit is $r = G\frac{M}{v^2}$.

(*b*) Momentum is conserved during the explosion, meaning that $p_i = (2m)v = m(2v) + m(v') = p_f$, where v' is the speed of the second piece after the explosion. This gives $v' = 0$. Hence, the second piece starts to fall down towards the planet with zero initial speed.

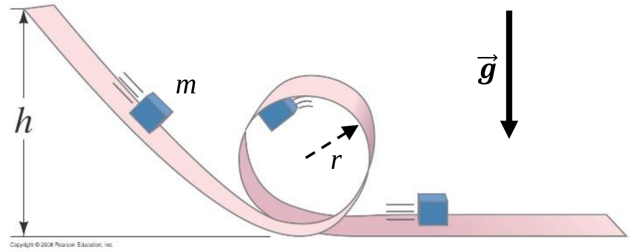
Since energy is conserved during the fall, and

$$E_i = -G\frac{Mm}{r} = -mv^2\,,\ E_f = \frac{1}{2}mv_f^2 - G\frac{Mm}{R}$$

we have

$$E_f = E_i \;\rightarrow\; \frac{1}{2}mv_f^2 - G\frac{Mm}{R} = -mv^2\text{, and }\ v_f = 2\sqrt{\frac{GM}{R} - v^2}\,.$$

3. The small mass m sliding without friction along the looped track shown in the figure is to remain on the track at all times, even at the very top of the loop of radius r.



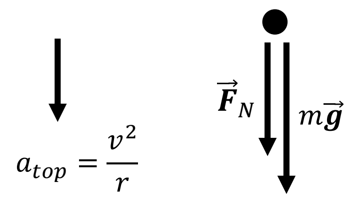
(a) (9 Pts.) In terms of the given quantities, determine the minimum release height h_{min}.

(b) (8 Pts.) If the actual release height is $h = 5r$, calculate the normal force exerted by the track at the top of the loop.

(c) (8 Pts.) What will be the speed of the block after it exits the loop onto the flat section if the release height is $h = 5r$?

Solution:

Free-body diagram of the mass at the top of the loop of radius r is



Therefore, Newton's second law gives

$F_N + mg = m\frac{v_{top}^2}{r}$, where v_{top} is the speed of the mass at the top of the loop. If the mass is to remain on the track at the top, the minimum value of the contact force at the top is zero. Therefore

$mg = m\frac{v_{top}^2}{r} \rightarrow v_{top} = \sqrt{gr}$ is the minimum speed required for the mass to remain on the track at the top.

$$E_i = mgh_{\min}, \; E_{top} = \frac{1}{2}mv_{top}^2 + 2mgr = \frac{5}{2}mgr .$$

Since the total mechanical energy is conserved, we have

$$mgh_{\min} = \frac{5}{2}mgr \rightarrow h_{\min} = \frac{5}{2}r .$$

(b) If the actual release height is $h = 5r$, we have $E_i = 5mgr$, $E_{top} = \frac{1}{2}mv_{top}^2 + 2mgr \rightarrow v_{top} = \sqrt{6gr}$.

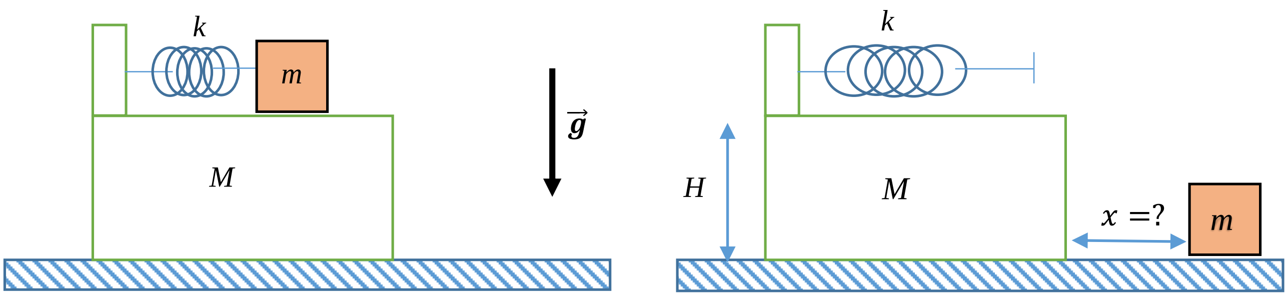
In this case $F_N + mg = m\frac{v_{top}^2}{r} \rightarrow F_N = 5mg$.

(c) $E_i = 5mgr$, $E_{flat} = \frac{1}{2}mv_{flat}^2 \rightarrow v_{flat} = \sqrt{10gr}$.

4. A mass m is on a platform of mass M and height H as shown in the figure. A spring with spring constant k, with one end fixed to the platform is used to launch the small mass horizontally. The platform is on a frictionless table, and the friction between the mass and the platform is also negligible. Initially the spring is compressed by an amount d from its natural length, and both the mass and the platform are at rest. Then the system is released.

(*a*) (12 Pts.) Find the velocities of the mass and of the platform at the instant the mass leaves the platform.

(b) (13 Pts.) Find the distance between the mass and the platform when the mass hits the floor.



Solution:

(*a*) Initial energy stored in the spring will transform into the kinetic energies of the block and the platform.

$\frac{1}{2}kd^2 = \frac{1}{2}mv^2 + \frac{1}{2}MV^2$, where v and V are velocities of the mass and the platform respectively.

Since momentum is conserved, we have $p_i = 0 = mv + MV = p_f \rightarrow V = -\frac{m}{M}v$. Therefore

$$\frac{1}{2}kd^2 = \frac{1}{2}mv^2 + \frac{1}{2}M\left(-\frac{m}{M}v\right)^2 \rightarrow v = d\sqrt{\frac{Mk}{m(M+m)}}, \text{ and } V = -d\sqrt{\frac{mk}{M(M+m)}}.$$

(*b*) The block falls a distance H before it hits the floor. This means $H = \frac{1}{2}gt^2 \rightarrow t = \sqrt{\frac{2H}{g}}$ is the time it takes the block to fall down. During this time the block travels a horizontal distance $d_{bl} = vt = v\sqrt{\frac{2H}{g}}$, while the platform travels a distance $d_{pl} = Vt = V\sqrt{\frac{2H}{g}}$ in the opposite direction. Therefore, the distance is

$$x = (v+V)t = \left(\sqrt{\frac{M}{m}} + \sqrt{\frac{m}{M}}\right)\left(\sqrt{\frac{k}{M+m}}\right)\left(\sqrt{\frac{2H}{g}}\right)d = d\sqrt{\frac{k(M+m)}{Mm}\frac{2H}{g}}.$$