PHYS 101 – General Physics I Midterm Exam 1

Duration: 120 minutes Saturday, 11 March 2017, 14:00

1. A car starts from rest and moves with constant acceleration $a = 2.0\ \mathrm{m/s^2}$ until it reaches $v = 26.0\ \mathrm{m/s}$ speed. It then continues to move with this constant speed.

(*a*) (8 Pts.) For how long does the car accelerate?

(*b*) (8 Pts.) How much distance is covered by the car during the time it accelerates?

(*c*) (9 Pts.) After reaching the speed of $v = 26.0\ \mathrm{m/s}$, how much longer should the car travel until its average speed becomes $\bar{v} = 24.0\,\mathrm{m/s}$?

Solution:

(*a*) For an object moving with constant acceleration, $v - v_0 = at$. Since the car starts from rest, we have $v_0 = 0$, and

$$t = \frac{v}{a} = \frac{26.0\,\mathrm{m/s}}{2.0\,\mathrm{m/s^2}} = 13.0\,\mathrm{s}\ .$$

(*b*) The position of an object moving in one dimension with constant acceleration is given by

$$x = x_0 + v_0 t + \frac{1}{2} a t^2 .$$

Therefore

$$x - x_0 = \frac{1}{2}(2.0\,\mathrm{m/s^2})(13.0\,\mathrm{s})^2 = 169.0\,\mathrm{m}.$$

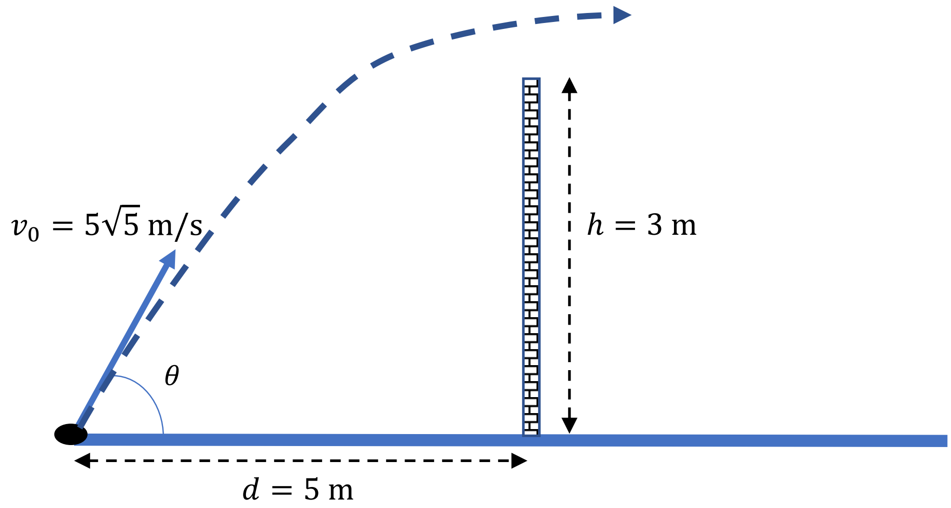
(*c*) Suppose that the car travels for T seconds after reaching the speed of $v = 26.0\ \mathrm{m/s}$. It's average speed will be

$$\bar{v} = \frac{\text{distance traveled}}{\text{time taken}} = \frac{(169.0\,\mathrm{m}) + (26.0\,\mathrm{m/s})T}{(13\mathrm{s}) + T}.$$

If the average speed is to be $\bar{v} = 24.0\,\mathrm{m/s}$, we must have

$$24.0\,\mathrm{m/s} = \frac{(169.0\,\mathrm{m}) + (26.0\,\mathrm{m/s})T}{(13\mathrm{s}) + T} \rightarrow T = \frac{143}{2}\mathrm{s} = 71.5\,\mathrm{s}.$$

2. (25 Pts.) A stone is to be thrown with speed $v_0 = 5\sqrt{5}\, m/s$ towards a thin vertical wall which has a height of $h = 3\, m$, and which is at a distance of $d = 5\, m$. Find the tangent of minimum and maximum angles of throw (i.e., find $\tan\theta$ for the angle θ shown in the figure) so that the stone can pass over the wall. (Take acceleration due to gravity as $10\, m/s^2$, and neglect air resistance.)



Solution:

The position of the projectile at any time will be given by

$$x(t) = v_{x0}t\,,\; y(t) = v_{y0}t - \frac{1}{2}g\,t^2\,.$$

$$x = (5\sqrt{5}\,\mathrm{m/s})\cos\theta t \quad \text{and} \quad y = (5\sqrt{5}\,\mathrm{m/s})\sin\theta t - (5\,\mathrm{m/s^2})t^2\,.$$

If the stone is to pass over the wall, we must have $y \geq 3$ m, when $x = 5$m at some time t. The time is found as

$$5\text{ m} = (5\sqrt{5}\,\mathrm{m/s})\cos\theta t \;\rightarrow\; t = \frac{1}{\sqrt{5}\cos\theta}\mathrm{s}\,.$$

Using this result in the expression for the coordinate *y*, we have

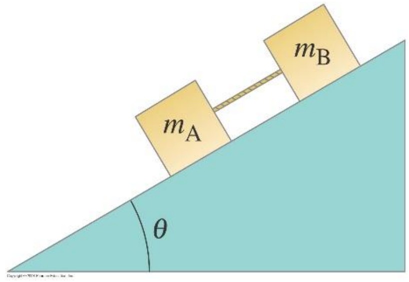
$$3 = (5\sqrt{5})\frac{\sin\theta}{\sqrt{5}\cos\theta} - (5)\left(\frac{1}{\sqrt{5}\cos\theta}\right)^2 \;\rightarrow 3 = 5\tan\theta - \sec^2\theta\,.$$

Using $\sec^2\theta = 1 + \tan^2\theta$, we get

$\tan^2\theta - 5\tan\theta + 4 = 0$, whose roots are $\tan\theta = 1$ and $\tan\theta = 4$. Therefore, for the stone to pass over the wall we must have

$$1 \leq \tan\theta \leq 4\,.$$

3. Two blocks of equal mass $m_A = m_B = m$, connected together by a massless cord of fixed length, slide down a plane ramp inclined at an angle θ to the horizontal as shown in the figure. The coefficient of kinetic friction between block A and the inclined surface is $\mu_A = \mu$, whereas the coefficient of kinetic friction between block B and the inclined surface is $\mu_B = 2\mu$.



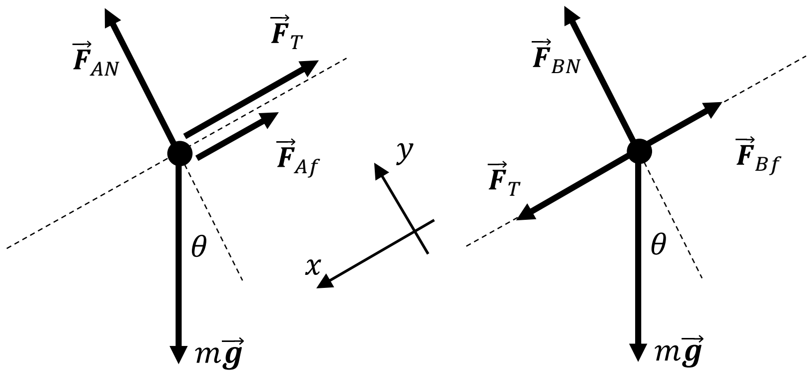
(*a*) (10 Pts.) Draw a free body diagram for each block.

(*b*) (10 Pts.) Find the acceleration of the blocks in terms of m, μ, θ and the gravitational acceleration *g*.

(*c*) (5 Pts.) Find the tension in the cord in terms of the above-mentioned parameters.

Solution:

(*a*)



(*b*) Because the blocks are connected together by a massless cord of fixed length, they will have the same acceleration down the inclined plane. Choosing the coordinate system as shown in the free body diagram, and writing Newton's second law for block A, we have

$$mg\sin\theta - F_T - F_{Af} = ma,\ F_{AN} - mg\cos\theta = 0.$$

Since $F_{AN} - mg\cos\theta = 0 \rightarrow F_{AN} = mg\cos\theta$, and $F_{Af} = \mu F_{AN} = \mu mg\cos\theta$, we have

$$mg\sin\theta - F_T - \mu mg\cos\theta = ma.$$

Similarly, noting that for block B we have $\mu_B = 2\mu$, we get

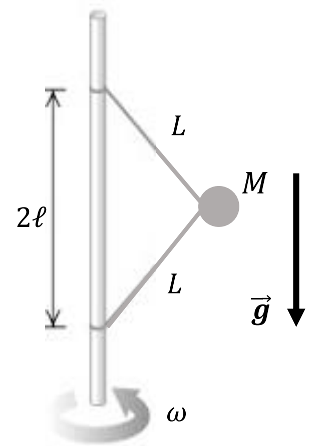
$$mg\sin\theta + F_T - 2\mu mg\cos\theta = ma.$$

Eliminating F_T between the two equations, we obtain $a = \left(\sin\theta - \frac{3}{2}\mu\cos\theta\right)g$.

(*a*) Using the expression for the acceleration found above in any one of the two equations, we find

$$F_T = \frac{1}{2}\mu mg\cos\theta.$$

4. A sphere of mass M is attached to a vertical rod by means of two strings, as shown in the figure. The system rotates about the axis of the rod with angular velocity ω, such that the strings are extended as shown in the diagram. For parts (a) and (b) the tension in the upper sting is twice the tension in the lower string.



(a) (7 Pts.) Draw a free body diagram for the sphere.

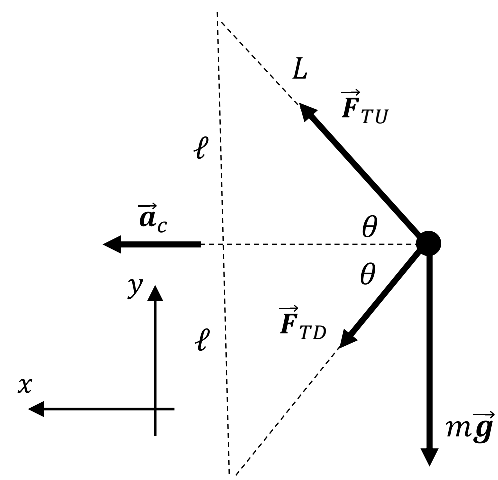
(b) (9 Pts.) What is the angular velocity ω of the system in terms of the other parameters?

(c) (9 Pts.) Find the angular velocity at which the lower cord just goes slack i.e., when the tension in the lower string becomes zero.

Solution:

(b) From the geometry of the figure, we see that

$$\sin\theta = \frac{\ell}{L} \quad \text{and} \quad \cos\theta = \frac{\sqrt{L^2-\ell^2}}{L} \; .$$



Choosing the coordinate system as shown in the free body diagram, and writing Newton's second law, we have

$$F_{TU}\cos\theta + F_{TD}\cos\theta = Ma_C = ML\cos\theta\,\omega^2$$

$$F_{TU}\sin\theta - F_{TD}\sin\theta - Mg = 0\,,$$

or

$$F_{TU} + F_{TD} = ML\omega^2 \quad \text{and} \quad F_{TU} - F_{TD} = \frac{Mg}{\sin\theta}$$. Solving these,

Given that $F_{TU} = 2F_{TD}$, we obtain

$$3F_{TD} = ML\omega^2 \quad \text{and} \quad F_{TD} = \frac{Mg}{\sin\theta}, \text{ meaning that } \frac{1}{3}ML\omega^2 = \frac{Mg}{\sin\theta} = \frac{LMg}{\ell} \;\rightarrow\; \omega = \sqrt{\frac{3g}{\ell}}\;.$$

(c) When the tension in the lower string becomes zero, we have $F_{TD} = 0$. The equations become

$$F_{TU} = ML\omega^2 \quad \text{and} \quad F_{TU} = \frac{Mg}{\sin\theta} = \frac{LMg}{\ell} \;\rightarrow\; ML\omega^2 = \frac{LMg}{\ell} \;\rightarrow\; \omega = \sqrt{\frac{g}{\ell}}\;.$$