PHYS 101 – General Physics I
Midterm Exam 2 Solultions

Saturday, 03 December 2016, 10:00 Duration: 120 minutes

1. Two satellites of equal mass *m* orbit the earth (mass M_E) in circular orbits with radii $r_1 = R$ and $r_2 = 2R$.

(*a*) (10 Pts.) Find the ratio of their speeds v_1/v_2.

(*b*) (12 Pts.) Calculate their total mechanical energies E_1, E_2, and find E_1/E_2.

(*c*) (3 Pts.) Which one is larger?

Solution:

(*a*) For satellites in circular orbit around the earth, we have

$$F_G = ma_c \quad \rightarrow \quad G\frac{M_E m}{r^2} = m\frac{v^2}{r} \quad \rightarrow \quad \frac{GM_E}{r} = v^2 .$$

Therefore, for satellites 1 and 2, we can write

$v_1^2 = \dfrac{GM_E}{R}$ and $v_2^2 = \dfrac{GM_E}{2R}$. This means $\dfrac{v_1^2}{v_2^2} = \dfrac{GM_E}{R}\dfrac{2R}{GM_E} = 2 \quad \rightarrow \quad \dfrac{v_1}{v_2} = \sqrt{2}$.

(*b*) $E = K + U = \dfrac{1}{2}mv^2 - G\dfrac{M_E m}{r}$.

But, for a satellite in a circular orbit, we also have $v^2 = \dfrac{GM_E}{r}$. Hence,

$$E = K + U = \frac{1}{2}m\frac{GM_E}{r} - \frac{GM_E m}{r} = -\frac{GM_E m}{2r} .$$

Therefore

$$E_1 = -\frac{GM_E m}{2R} \quad \text{and} \quad E_2 = -\frac{GM_E m}{4R} .$$

This means

$$\frac{E_1}{E_2} = \left(-\frac{GM_E m}{2R}\right)\left(\frac{-4R}{GM_E m}\right) \quad \rightarrow \quad \frac{E_1}{E_2} = 2 .$$

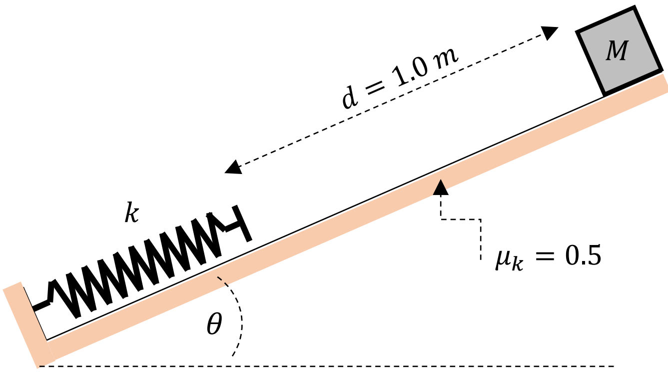
(c) $E_1 < E_2$ because $-\dfrac{GM_E m}{2R} < -\dfrac{GM_E m}{4R}$.

2. A box of mass $M = 0.5\ kg$ starts from rest and slides $d = 1.0\ m$ down an inclined plane before hitting a spring. Assume that $g = 10\ m/s^2$, the coefficient of kinetic friction between the box and the surface is $\mu_k = 0.5$, and $\sin\theta = 3/5$.

(*a*) (8 Pts.) What is the speed of the box just before it hits the spring?

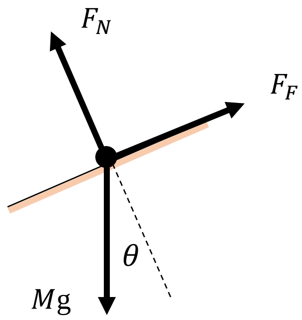
(*b*) (9 Pts.) If the box compresses the spring $\Delta x = 0.2\ m$ from its equilibrium position before coming to rest, what is the stiffness constant of the spring?

(*c*) (8 Pts.) We observe that the compressed spring can push the block back up the inclined plane after it stops momentarily. What is the maximum value of the coefficient of static friction μ_s betweeen the box and the surface?



Solution:

Since there is friction, the total mechanical energy is not conserved. We have $\Delta E = E_f - E_i = W_F$, where W_F represents work done by the friction force. From the free-body diagram, we see that



$F_N - Mg\cos\theta = 0 \ \rightarrow\ F_N = Mg\cos\theta$. Since for the kinetic case, we have

$F_F = \mu_k F_N \ \rightarrow\ F_F = \mu_k Mg\cos\theta$.

(*a*) Take the zero gravitational potential energy level as the initial position of the box potential. Just before the box hits the spring, we have

$\Delta E = \frac{1}{2}Mv_f^2 - Mgd\sin\theta = W_F = -\mu_k Mg\cos\theta\, d$. Therefore

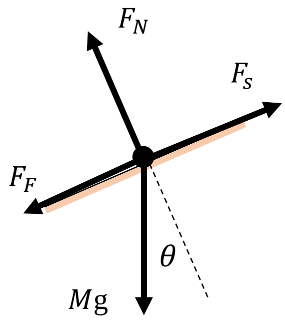
$v_f^2 = 2gd(\sin\theta - \mu_k\cos\theta) = 2(10\,m/s^2)(1m)\left(\frac{3}{5} - \frac{1}{2}\frac{4}{5}\right) = 4m^2/s^2$. Hence. $v_f = 2m/s$.

(*b*) The box comes to rest after compressing the spring, therefore

$\Delta E = \frac{1}{2}k(\Delta x)^2 - Mg(d+\Delta x)\sin\theta = W_F = -\mu_k Mg\cos\theta(d+\Delta x)$. Therefore,

$$k = \frac{2Mg(d+\Delta x)}{(\Delta x)^2}(\sin\theta - \mu_k\cos\theta) = 60\,N/m$$

(*c*) If the box does not move after compressing the spring, we have

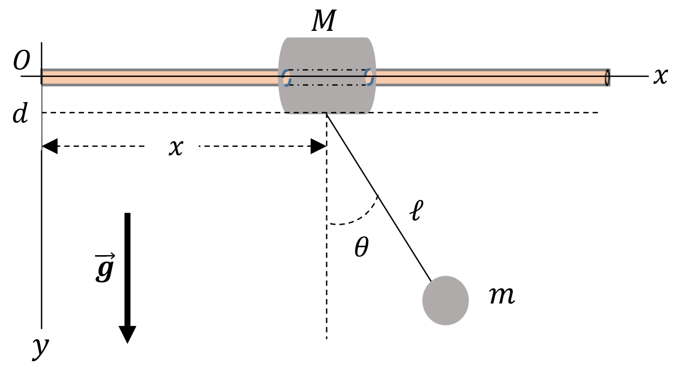


$F_F + Mg\sin\theta = |F_s| = k\,\Delta x$. Since for the static case

$F_F \le \mu_s F_N \ \rightarrow\ \mu_s Mg\cos\theta \ge k\,\Delta x - Mg\sin\theta$, which gives

$4\mu_s \ge 12 - 3 \ \rightarrow\ \mu_s \ge \frac{9}{4} = 2.25$. Since the spring can push the block back up, we must have $\mu_{s\max} = \frac{9}{4} = 2.25$.

3. A cylinder of mass *M* is free to slide on a frictionless horizontal shaft passing through its axis. A ball of mass *m* is attached to the cylinder by a massless string of length ℓ. Initially, both the cylinder and the ball are at rest, with the center of the cylinder at a perpendicular distance x_0 from the *y*-axis, and the ball displaced by an angle $\theta_0 = \pi/2$ to the right relative to the vertical. Use the coordinate system indicated in the figure and assume that the motion takes place on the *xy*- plane.



(*a*) (10 Pts.) If the ball is released from its initial position $(x_0 + \ell, d)$ with zero initial velocity, what will be its position when it is at the bottom of its swing, i.e., when $\theta = 0$?

(*b*) (15 Pts.) Find the velocities of the ball and the cylinder when $\theta = 0$.

Solution:

(*a*) Initially, the center of mass of the system is at rest and its *x*-component is $X_{CM} = \dfrac{Mx_0 + m(x_0+\ell)}{M+m}$.

X_{CM} does not change during the swinging motion of the ball because external forces, i.e., force of gravity and the supporting force of the shaft, are in the *y*-direction. Therefore, when the ball swings down to lowest position, we have

$$X_{CM} = \frac{Mx' + mx'}{M+m} = \frac{Mx_0 + m(x_0+\ell)}{M+m} \quad \rightarrow \quad x' = \frac{Mx_0 + m(x_0+\ell)}{M+m}.$$

The position of the ball will therefore be

$$x' = x_0 + \frac{m\ell}{M+m}, \; y' = \ell + d\ .$$

(*b*) The *x*-component of the momentum of the system is conserved. Furthermore, at the bottom of its swing the ball's velocity will be in the $-x$ direction, while the cylinder's velocity will be in the $+x$ direction. Since the initial momentum is zero, when the ball is at the bottom of its swing we have

$$MV - mv = 0,$$

where *V* and -*v* are *x*-components of the velocities of the cylinder and the ball respectively.

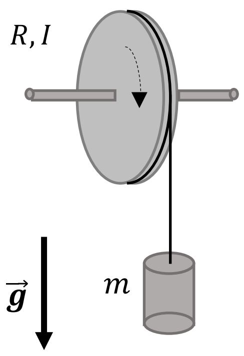
Total mechanical energy is also conserved. Taking $y = d$ as the zero level for gravitational potential energy, we have

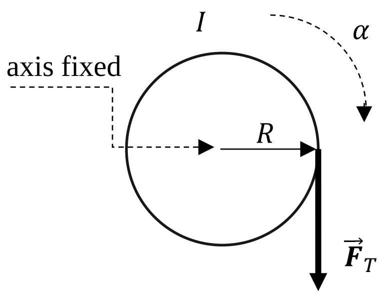
$$E_f = \frac{1}{2}MV^2 + \frac{1}{2}mv^2 - mg\ell = 0.$$

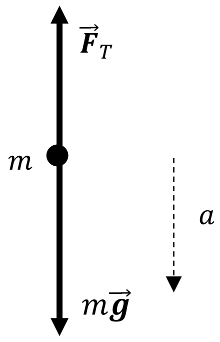
Solving these two equations for *V* and *v*, we get

$$v = \sqrt{\frac{2Mg\ell}{M+m}}, \; V = \frac{m}{M}v = \sqrt{\frac{2m^2g\ell}{M(M+m)}}\ .$$

4. (25 Pts.) A wheel of radius R and moment of inertia I is mounted on a stationary frictionless horizontal axle as shown in the figure. A light chord wrapped around the wheel supports an object of mass m. The system is released from rest and the object starts falling down turning the wheel. Assuming that the chord does not slip on the wheel, calculate the acceleration of the object of mass m, and the tension in the chord. (Draw free-body diagrams first.)







Newton's second law applied to the rotational motion of the wheel around its fixed axis gives

$$\tau = RF_T = I\alpha \, .$$

Newton's second law applied to the translational motion of the object gives

$$mg - F_T = ma \, .$$

If the chord does not slip on the wheel, we have $a = R\alpha$. Therefore,

$$F_T = \frac{I\alpha}{R} \;\rightarrow\; mg - \frac{I\alpha}{R} = ma \;\rightarrow\; mg - \frac{Ia}{R^2} = ma \, .$$

Solving for a, we get

$$mg = \left(\frac{I}{R^2} + m \right) a \;\rightarrow\; a = \frac{mR^2 g}{I + mR^2} \, ,$$

and

$$F_T = \frac{I\alpha}{R} = \frac{Ia}{R^2} \;\rightarrow\; F_T = \frac{m I g}{I + mR^2} \, .$$