PHYS 101 – General Physics I

Midterm Exam 1 Solulutions

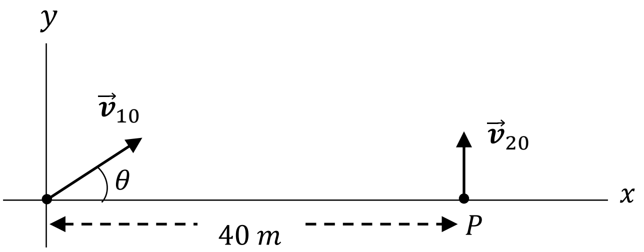
Saturday, 22 October 2016, 10:00 Duration: 120 minutes

1. A projectile is shot from the origin with an initial speed of $v_{10} = 25\ m/s$, at an angle θ with the horizontal. At the same time, a second projectile is shot from the point P, $(x_p = 40\ m, y_p = 0\ m)$ vertically up with an initial speed of $v_{20} = 15\ m/s$. Assume that the acceleration due to gravity is $g = 10\ m/s^2$, and that both projectiles are shot at time $t = 0$.

(*a*) (9 Pts.) Write the kinematical equations describing the positions $\vec{r}_1(t)$ and $\vec{r}_2(t)$ of each projectile as a function of time *t* using the coordinate system given in the figure.

(*b*) (8 Pts.) If the two projectiles are to hit each other, what must $\sin\theta$ be?

(*c*) (8 Pts.) At what height *h* from the ground do the projectiles hit each other?



Solution:

(*a*) The position of an object moving with constant acceleration is given by $\vec{\mathbf{r}}(t) = \vec{\mathbf{r}}_0 + \vec{\mathbf{v}}_0 t + \frac{1}{2}\vec{\mathbf{a}}t^2$.

For the first projectile, we have $\vec{\mathbf{r}}_{10} = 0$, $\vec{\mathbf{v}}_{10} = (25\ m/s)\cos\theta\,\hat{\mathbf{i}} + (25\ m/s)\sin\theta\,\hat{\mathbf{j}}$ and $\vec{\mathbf{a}} = -g\,\hat{\mathbf{j}}$, while for the second projectile, we have $\vec{\mathbf{r}}_{20} = (40\,m)\,\hat{\mathbf{i}}$, $\vec{\mathbf{v}}_{20} = (15\ m/s)\,\hat{\mathbf{j}}$ and $\vec{\mathbf{a}} = -g\,\hat{\mathbf{j}}$.

Using these, we find:

$$\vec{\mathbf{r}}_1(t) = \left[(25\ m/s)\cos\theta\, t\right]\hat{\mathbf{i}} + \left[(25\ m/s)\sin\theta\, t - (5\,m/s^2)t^2\right]\hat{\mathbf{j}}\ .$$

Components are

$$x_1(t) = (25\ m/s)\cos\theta\, t,\ y_1(t) = (25\ m/s)\sin\theta\, t - (5\,m/s^2)t^2\ .$$

Similarly, for the second projectile, we find:

$$\vec{\mathbf{r}}_2(t) = \left[(40\ m)\right]\hat{\mathbf{i}} + \left[(15\ m/s)t - (5\,m/s^2)t^2\right]\hat{\mathbf{j}}\ .$$

$$x_2 = 40\ m,\ y_2(t) = (15\ m/s)t - (5\,m/s^2)t^2\ .$$

(*b*) If the two projectiles are to hit each other, they must be at the same point at the same time. Therefore

$$\vec{\mathbf{r}}_1(t) = \vec{\mathbf{r}}_2(t)\ \rightarrow\ (40\ m) = (25\ m/s)\cos\theta\, t \text{ and } (15\ m/s)t - (5\,m/s^2)t^2 = (25\ m/s)\sin\theta\, t - (5\,m/s^2)t^2$$

From the second equation , we get $(15\ m/s) = (25\ m/s)\sin\theta\ \rightarrow\ \sin\theta = \frac{3}{5}$.

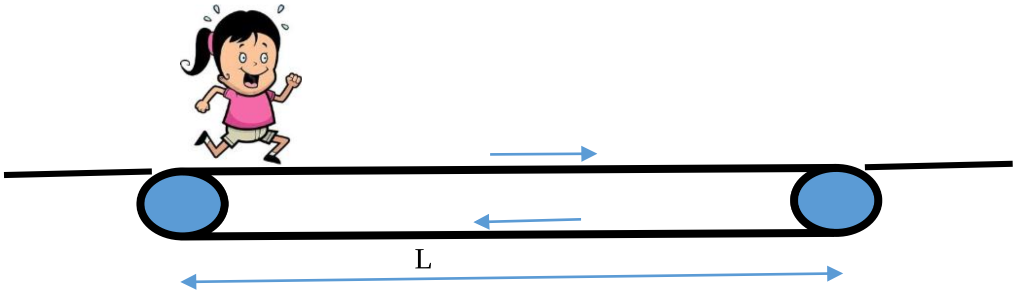
$$\sin\theta = 3/5$$

This means $\cos\theta = \frac{4}{5}$, and the first equation gives $(40\ m) = (25\ m/s)\cos\theta\, t_c\ \rightarrow\ t_c = 2\,s$ for the time of collision, when both projectiles are at height $y_1(2) = y_2(2) = 10\,m$.

$$h = 10\ m$$

2. (20 Pts.) A child runs on a moving walkway of length L at the airport. If she runs in the direction of motion of the walkway she crosses the walkway in t_1 seconds. If she runs in the opposite direction she crosses the walkway in t_2 seconds. Assume that she always runs with the same constant velocity with respect to the walkway.

If she does not run but stands still on the walkway how long would it take for her to cross it?



Solution:

When the child is running in the direction of motion of the walkway, her speed with respect to the earth v_{ce} is $v_{ce} = v_{cw} + v_{we}$, where v_{cw} is the speed of the child with respect to the walkway, and v_{we} is the speed of the walkway with respect to the earth. This means

$$L = (v_{cw} + v_{we})t_1 .$$

When the child is running in the opposite direction of motion of the walkway, her speed with respect to the earth becomes $v_{ce} = v_{cw} - v_{we}$. Notice that to be able to cross the distance *L*, we must have $v_{cw} > v_{we}$. In this case

$$L = (v_{cw} - v_{we})t_2 ,$$

and we have $t_2 > t_1$. If the child does not run but stands still on the walkway, her speed with respect to the earth becomes $v_{ce} = v_{we}$, hence, in this case

$$L = v_{we}\, t \;\rightarrow\; t = \frac{L}{v_{we}} .$$

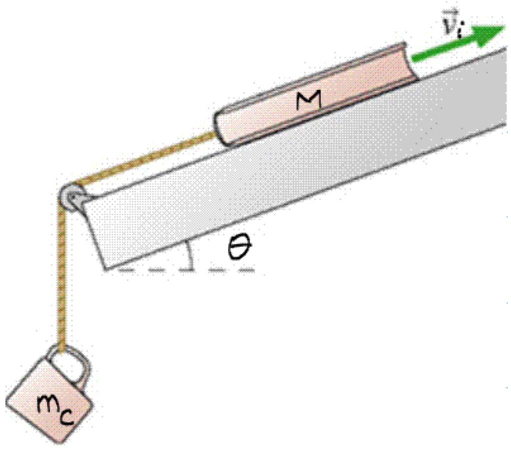
From first two equations, we get

$$v_{cw} + v_{we} = \frac{L}{t_1},\; v_{cw} - v_{we} = \frac{L}{t_2} \;\rightarrow\; 2v_{we} = \frac{L}{t_1} - \frac{L}{t_2} \;\rightarrow\; v_{we} = \frac{L(t_2 - t_1)}{2t_1 t_2} .$$

Hence, we have

$$t = \frac{2t_1 t_2}{t_2 - t_1}$$

3. A physics book of mass M is connected by an unstrectchable string over a pulley to a dangling coffee cup of mass m_c. The book is given a push up the inclined plane, which makes an angle θ with the horizontal, and is released with an initial speed v_i, as shown in the figure. The coefficient of kinetic friction between the inclined plane and the book is μ_k.



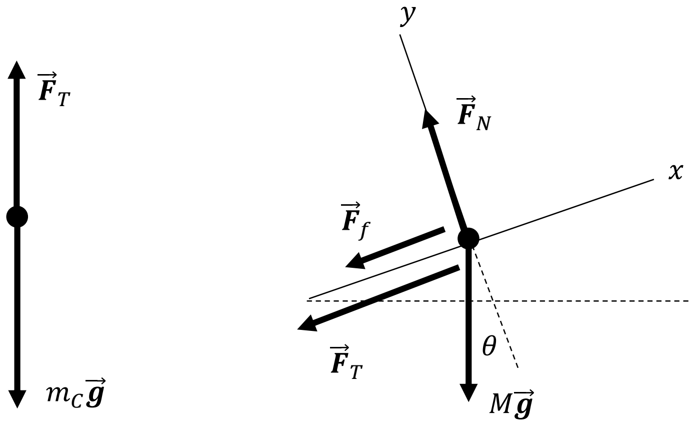
Aswer the following questions expressing your answers in terms of $\{v_i, M, m_c, \theta, g, \mu_k\}$. (Dont use the work-energy theorem!)

(a) (10 Pts.) Draw the free body diagrams for the book and the cup.

(b) (15 Pts.) What distance d does the book slide on the inclined plane after being released before coming to a stop temporarily? How much time T does it take for the book to stop after being released?

Solution:

(a)



(b) Since the book and the cup are connected by an unstrectchable string, the magnitude of their accelerations will be the same. Writing Newton's second law for the two objects, we have

$F_T - m_C g = m_C a$, $-F_T - F_f - Mg\sin\theta = Ma$ and $F_N - Mg\cos\theta = 0$. We also have $F_f = \mu_k F_N$.

These mean $F_N = Mg\cos\theta \rightarrow F_f = \mu_k Mg\cos\theta$ and $F_T = m_C(g+a)$. Using these, and solving the second equation for the acceleration, we get

$$a = -\left(\frac{M\sin\theta + \mu_k M\cos\theta + m_C}{M + m_C}\right) g \, .$$

The distance d the book slides on the inclined plane after being released and before coming to a momentary stop can be found using the relation $v_f^2 - v_i^2 = 2\,a\,d$, with $v_f = 0$. Using the result for the acceleration, we find $T = \dfrac{-v_i^2}{2a}$

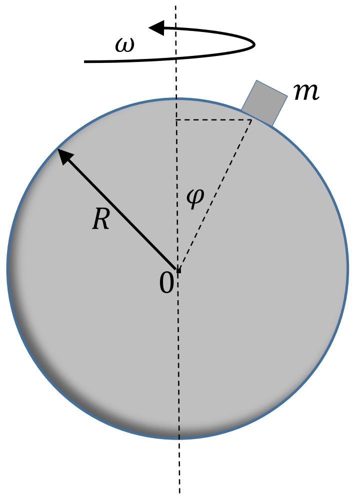
$$d = \frac{(M + m_C)v_i^2}{2(M\sin\theta + \mu_k M\cos\theta + m_C)g}$$

The time T it takes for the book to stop after being released is found using the relation $v_f = v_i + aT$. With $v_f = 0$, we have

$$T = \frac{-v_i}{a} \, .$$

$$T = \frac{(M + m_C)v_i}{(M\sin\theta + \mu_k M\cos\theta + m_C)g}$$

4. A small mass m is put on the surface of the sphere of radius R as shown in the figure. The coefficient of static friction between the mass and the surface is μ_s. Aswer the following questions expressing your answers in terms of $\{R, m, \varphi, g, \mu_s\}$.



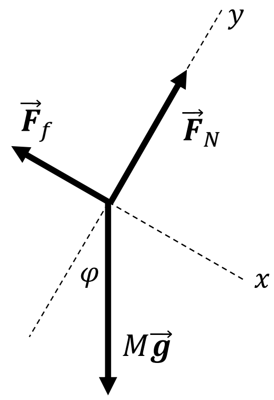
(a) (10 Pts.) Initially the sphere and the mass are at rest. Find the minimum value of μ_s so that the mass does not slide on the sphere because of friction?

(b) (20 Pts.) If the sphere slowly starts to rotate with increasing angular speed about the vertical axis passing through its center 0, what will the angular speed ω of the sphere be when the mass begins to slide?

Solution:

(a) When the sphere and the mass are at rest, we have

$$Mg\sin\varphi - F_f = 0,\ \ Mg\cos\varphi - F_N = 0\ .$$



Hence:

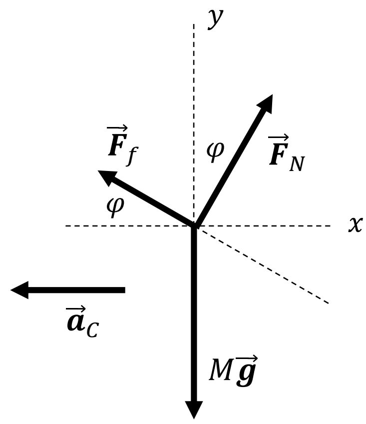
$F_f = Mg\sin\phi$ and $F_N = Mg\cos\varphi$. Since

$F_f \le \mu_s F_N$, we have $Mg\sin\phi \le \mu_s Mg\cos\varphi$, or

$\tan\phi \le \mu_s$. This means

$$\boxed{\mu_{s(\min)} = \tan\varphi}$$

(b) When the sphere starts to rotate, the mass will start to rotate around a circle with radius $R\sin\varphi$, and therefore, will have an acceleration $\vec{a}_C$ (centripetal) directed towards the axis of rotation. Decomposing the forces along the directions shown and writing Newton's second law, we have



$$F_f\cos\varphi - F_N\sin\varphi = m a_c\ ,\ F_f\sin\varphi + F_N\cos\varphi - m g = 0\,.$$

These two equations can be solved for F_f and F_N. Multiplying the first equation by $\cos\varphi$, the second equation by $\sin\varphi$, and adding the resulting equations, we find $F_f = m a_c\cos\varphi + m g\sin\varphi$. Similarly, multiplying the first equation by $\sin\varphi$, the second equation by $\cos\varphi$, and subtracting the resulting equations, we find $F_N = -m a_c\sin\varphi + m g\cos\varphi$.

Hence, the condition $F_f \le \mu_s F_N$ implies

$(m a_c\cos\varphi + m g\sin\varphi) \le \mu_s(-m a_c\sin\varphi + m g\cos\varphi)$. Solving for a_C and using the fact that the small mass rotates around a circle with radius $R\sin\varphi$, we have $a_C = R\sin\varphi\ \omega^2$, and

$$\omega^2 \le \frac{g}{R\sin\varphi}\left(\frac{\mu_s\cos\varphi - \sin\varphi}{\mu_s\sin\varphi + \cos\varphi}\right).$$

$$\boxed{\omega \le \sqrt{\frac{g}{R}\left(\frac{\mu_s\cos\varphi - \sin\varphi}{\mu_s\sin^2\varphi + \cos\varphi\sin\varphi}\right)}}$$