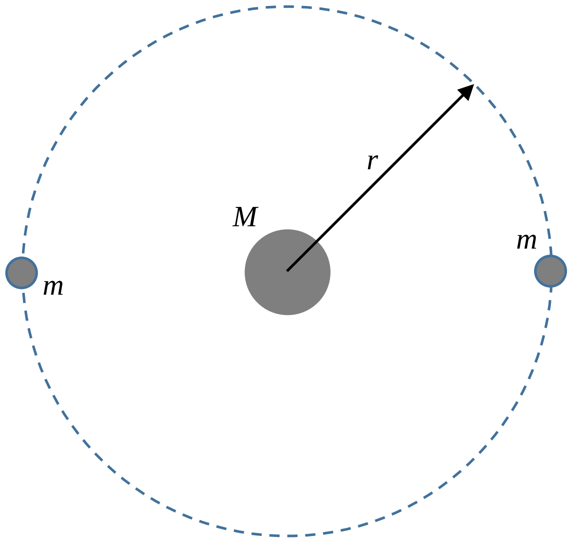
PHYS 101 Midterm Exam 2 (09.04.2016) Solutions

1. Consider a star with two identical planets in the same circular orbit with radius *r* encircling the star with exactly the same period *T*, at opposite ends of a diameter. The mass of the star is twenty times the mass of each planet ($M = 20\, m$).



(a) (10 Pts.) Find the magnitude of the total gravitational force exerted on one of the planets.

(b) (15 Pts.) Find the period of the planets in terms of the mass *m* and the radius *r* of the orbit.

Solution:

(a) $F_{total} = G\frac{Mm}{r^2} + G\frac{m^2}{4r^2} = G\frac{m}{r^2}\left(M + \frac{m}{4}\right)$

Since $M = 20m$, we get

$$F_{total} = G\frac{m^2}{r^2}\left(20 + \frac{1}{4}\right) = G\left(\frac{9m}{2r}\right)^2 .$$

(b) For a circular orbit, the force of gravity is the centripetal force. Therefore

$G\left(\frac{9m}{2r}\right)^2 = m r \omega^2 \rightarrow \omega = \frac{9m}{2r}\sqrt{\frac{G}{mr}} = \frac{9}{2}\sqrt{\frac{Gm}{r^3}}$. Since $T = \frac{2\pi}{\omega}$, we obtain

$$T = \frac{4\pi}{9}\sqrt{\frac{r^3}{Gm}} .$$

2. Two forces, $\vec{\mathbf{F}}_1 = (1.5\,N)\,\hat{\mathbf{i}} - (0.8\,N)\,\hat{\mathbf{j}} + (0.7\,N)\,\hat{\mathbf{k}}$ and, $\vec{\mathbf{F}}_2 = (0.5\,N)\,\hat{\mathbf{i}} - (0.2\,N)\,\hat{\mathbf{j}} - (0.7\,N)\,\hat{\mathbf{k}}$, are applied on a moving object with mass $m = 0.5\,kg$. The displacement of the object under the action these forces is given by $\vec{\mathbf{d}} = (8m)\,\hat{\mathbf{i}} + (4m)\,\hat{\mathbf{j}} + (6m)\,\hat{\mathbf{k}}$.

(a) (8 Pts.) Fid the net work done on the object by the two forces.

(b) (8 Pts.) If it takes $\Delta t = 2\,s$ for the object to be displaced by $\vec{\mathbf{d}}$, how much average power was delivered to the object by the net force?

(c) (9 Pts.) If the speed of the object is $v = 4\,m/s$ at the initial point of the displacement $\vec{\mathbf{d}}$, what will the speed be at the final point of the displacement?

Solution:

(a) The net force on the object is $\vec{\mathbf{F}}_{net} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 = (2.0\,N)\,\hat{\mathbf{i}} - (1.0\,N)\,\hat{\mathbf{j}}$. Work done by the net force is

$$W = \vec{\mathbf{F}}_{net} \cdot \vec{\mathbf{d}} = \left[(2.0\,N)\,\hat{\mathbf{i}} - (1.0\,N)\,\hat{\mathbf{j}}\right] \cdot \left[(8m)\,\hat{\mathbf{i}} + (4m)\,\hat{\mathbf{j}} + (6m)\,\hat{\mathbf{k}}\right] = (16-4)J = 12\,J\ .$$

(b) $P_{average} = \dfrac{W}{\Delta t} = \dfrac{12\,J}{2\,s} = 6W$.

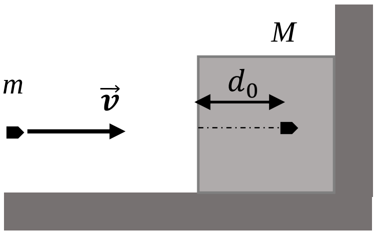
(c) According to the work – energy theorem $\Delta K = K_{fin} - K_{in} = W$. Therefore,

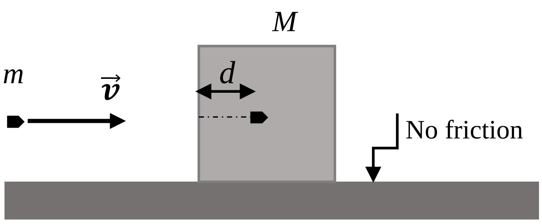
$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = W \;\rightarrow\; v_f = \sqrt{v_i^2 + \frac{2}{m}W}\ .$$

$$v_f = \sqrt{(4\,m/s)^2 + \frac{2}{(0.5\,kg)}(12\,J)} = \sqrt{(16\,m^2/s^2) + (48\,J/kg)} = \sqrt{(64\,m^2/s^2)} = 8\,m/s\ .$$

$$v_f = 8\ m/s\ .$$

3. (25 Pts.) A plastic block with mass *M* is on a frictionless horizontal surface resting against a wall, as shown in the first figure below. A bullet with mass *m* is shot into it with speed v, and it is observed that the block does not move as the bullet penetrates a distance d_0 into the block before stopping. Assuming that a constant friction force between the plastic and the bullet stops the bullet, find the distance *d* the bullet would penetrate in term of *M*, *m* and d_0, if there had been no wall to rest against (second figure below).





Solution:

If a constant friction force F_f stops the bullet when the block is resting against the wall, we have

$\Delta K = K_f - K_i = W_f$ by the work – energy theorem, where W_f is the work done by the constant friction force F_f. This means

$0 - \frac{1}{2}mv^2 = -F_f\, d_0$. Solving for F_f, we find the expression for the friction force as $F_f = \frac{mv^2}{2d_0}$.

If the block is not resting against the wall, we have an inelastic collision between the bullet and the block, after which the bullet becomes imbedded in the block. Since momentum of the system is conserved, we have

$mv = (m+M)v_f$, where v_f is the velocity of the block with the bullet inside, after the collision. Solving for v_f, we find $v_f = \left(\frac{m}{m+M}\right)v$.

The kinetic energy lost in the collision is found as

$$\Delta K = \frac{1}{2}(m+M)\left(\frac{m}{m+M}\right)^2 v^2 - \frac{1}{2}mv^2 = \left(\frac{m}{m+M} - 1\right)\frac{1}{2}mv^2 = -\left(\frac{M}{m+M}\right)\frac{1}{2}mv^2.$$

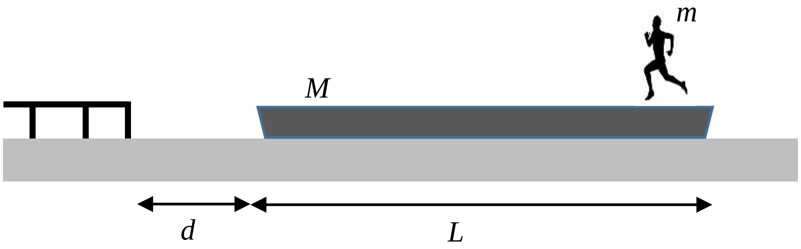
By the work – energy theorem, this energy loss must be equal to the work done by the friction force. So we have

$-\left(\frac{M}{m+M}\right)\frac{1}{2}mv^2 = -\frac{mv^2}{2d_0}d$. Hence, we get $d = \left(\frac{M}{m+M}\right)d_0$.

4. A person with mass m is at the back end of a flatboat with mass M and length L floating on water. Initially, the person and the flatboat are at rest and the distance from the front end of the flatboat to the pier is d. The person stars to run towards the pier with speed v_r relative to the flatboat. Assume that the boat moves without friction on water.

(a) (10 Pts.) What is the velocity of the runner with respect to the pier in terms of m, M and v_r?

(b) (10 Pts.) What will be the distance between the flatboat and the pier when the runner reaches the front end?



(c) (5 Pts.) If the runner suddenly stops running at the front end of the flatboat, what will be the speed of the flatboat relative to the pier?

Solution:

(a) Initially, both the boat and the person is at rest. So, the total momentum of the system is zero. When the person starts running towards the pier, the boat will start to move away from the pier to conserve momentum. Hence, we'll have

$$p_i = p_f \;\rightarrow\; 0 = m v_{RP} + M v_{BP},$$

where v_{RP} is the velocity of the runner with respect to the pier, and v_{BP} is the velocity of the boat with respect to the pier. Since $v_{RB} + v_{BP} = v_{RP}$, and $v_{RB} = v_r$, we have $v_r + v_{BP} = v_{RP} \;\rightarrow\; v_{BP} = v_{RP} - v_r$. Therefore

$$m v_{RP} + M(v_{RP} - v_r) = 0 \;\rightarrow\; v_{RP} = \frac{M\, v_r}{m+M}.$$

(b) The center of mass of the system (boat + person) will not move during the run. Therefore,

$$x_{CM} = (d+L)m + (d + \frac{L}{2})M = d'm + (d' + \frac{L}{2})M ,$$

where d' is the distance between the boat and the pier when the runner reaches the front end. Hence

$$d\,m + L\,m + d\,M + \frac{L}{2}M = d'\,m + d'\,M + \frac{L}{2}M \;\rightarrow\; L\,m = (d'-d)(m+M),$$ and the result becomes

$$d' - d = \frac{m\,L}{m+M} .$$

(c) Zero. Because the momentum is conserved.