Phys 101 – General Physics I Midterm Exam 1 Solutions

Saturday, 05 March 2015, 14:00

1. A projectile is shot horizontally from a cliff 20 *m* above sea level with an initial velocity 8 *m/s*. Using the coordinate system shown in the figure and taking $g = 10\ m/s^2$, find,

(a) (5 Pts.) the time it takes the projectile to hit the water surface,

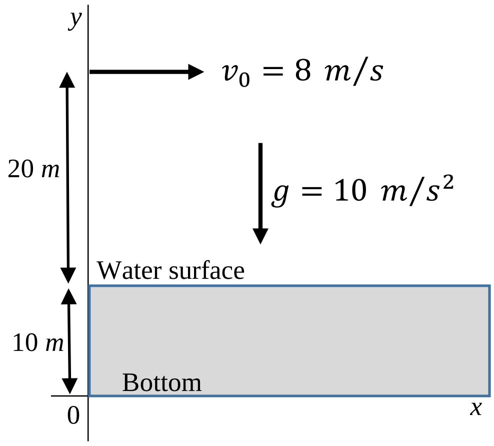
(b) (5 Pts.) The point at which the projectile hits the water surface,

(c) (5 Pts.) The velocity of the projectile at the time it hits the water surface.

After it enters into the water, the acceleration of the projectile is given by the vector $\vec{\mathbf{a}} = (-2\,m/s^2)\,\hat{\mathbf{i}} + (20\,m/s^2)\,\hat{\mathbf{j}}$.

(d) (5 Pts.) Find the time it takes for the projectile to reach the bottom, which is 10 *m* deep.

(e) (5 Pts.) Where does the projectile finally land?



Solution: Equation describing the motion of the projectile is $\vec{\mathbf{r}}(t) = \vec{\mathbf{r}}_0 + \vec{\mathbf{v}}_0\,t + \frac{1}{2}\vec{\mathbf{a}}\,t^2$. With the given initial conditions, the components are:

$x(t) = (8\,m/s)t$ and $y(t) = (30\,m) - (5\,m/s^2)t^2$.

(a) The projectile hits the water surface means $y = 10\,m$. So, $y(t) = (30\,m) - (5\,m/s^2)t^2 = 10\,m \rightarrow t = 2\,s$.

(b) When $t = 2\,s$, we have $x(2) = (8\,m/s)(2\,s) = 16\,m$. Therefore, the projectile hits the water surface at the point $x = 16\,m, y = 10\,m$.

(c) Components of the velocity of the projectile at any time is $v_x(t) = (8\,m/s)$, $v_y(t) = -(10\,m/s^2)t$. At time $t = 2\,s$ (when it hits the water surface), its velocity is $\vec{\mathbf{v}}(2) = (8\,m/s)\,\hat{\mathbf{i}} - (20\,m/s)\,\hat{\mathbf{j}}$.

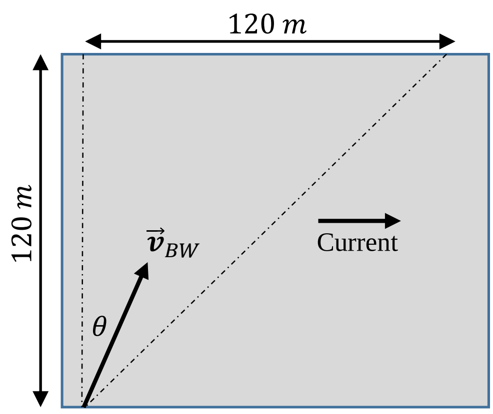
(d) Setting $t = 0$ when the projectile hits the water surface, the motion of the projectile in the water is described by the equation

$$\vec{\mathbf{r}}(t) = [(16\,m) + (8\,m/s)t + \frac{1}{2}(-2\,m/s^2)t^2]\,\hat{\mathbf{i}} + [(10\,m) + (-20\,m/s)t + \frac{1}{2}(20\,m/s^2)t^2]\,\hat{\mathbf{j}}$$

The projectile hits the bottom means $y = 0$. So
$(10\,m) - (20\,m/s)t + (10\,m/s^2)t^2 = 0 \rightarrow t^2 - 2t + 1 = 0 \rightarrow t = 1\,s$.

(e) When $t = 1\,s$, the position of the particle is $\vec{\mathbf{r}}(1) = [23\,m]\,\hat{\mathbf{i}} + [0]\,\hat{\mathbf{j}}$.

2. A boat, whose speed in still water is 2.0 m/s, must cross a 120 m – wide channel and arrive at a point 120 m downstream from where it starts. To do so, the captain should head the boat at an angle θ with the vertical as shown in the figure. Given that $\sin\theta = 3/5$.



(a) (15 Pts.) What is the speed of the current?

(b) (10 Pts.) How long does it take the boat to cross the channel?

Solution:

(a) Taking the horizontal direction to the right as *x* and the vertical direction upward as *y*, velocity of the boat with respect to the water is

$$\vec{\mathbf{v}}_{BW} = (2.0\,m/s)(\sin\theta\,\hat{\mathbf{i}} + \cos\theta\,\hat{\mathbf{j}}) = (2.0\,m/s)(\frac{3}{5}\hat{\mathbf{i}} + \frac{4}{5}\hat{\mathbf{j}}) = (\frac{6}{5}m/s)\hat{\mathbf{i}} + (\frac{8}{5}m/s)\hat{\mathbf{j}}$$

Velocity of the water (current) with respect to the shore is

$$\vec{\mathbf{v}}_{WS} = v_c\,\hat{\mathbf{i}}\,.$$

The velocity of the boat with respect to the shore is given by

$$\vec{\mathbf{v}}_{BS} = \vec{\mathbf{v}}_{BW} + \vec{\mathbf{v}}_{WS}\,,$$

therefore

$$\vec{\mathbf{v}}_{BS} = (v_c + \frac{6}{5}m/s)\hat{\mathbf{i}} + (\frac{8}{5}m/s)\hat{\mathbf{j}}.$$

To reach the destination we must have

$$v_c + \frac{6}{5}m/s = \frac{8}{5}m/s \;\rightarrow\; v_c = \frac{2}{5}m/s\,.$$

(b) To cross the channel, the boat must cover a vertical distance of $120\,m$ with the vertical speed of $\frac{8}{5}m/s$.

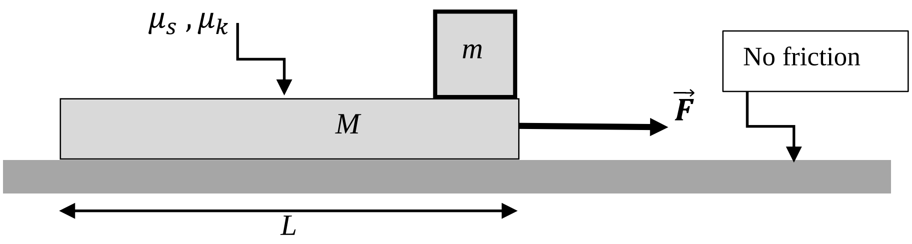
Therefore, time taken will be

$$t = \frac{120\,m}{(8/5\,m/s)} = 75\,s\,.$$

3. A small box of mass *m* is sitting on a board of mass *M* and length *L*. The board rests on a frictionless horizontal surface. Coefficients of static and kinetic friction between the board and the box is μ_s and μ_k, respectively.

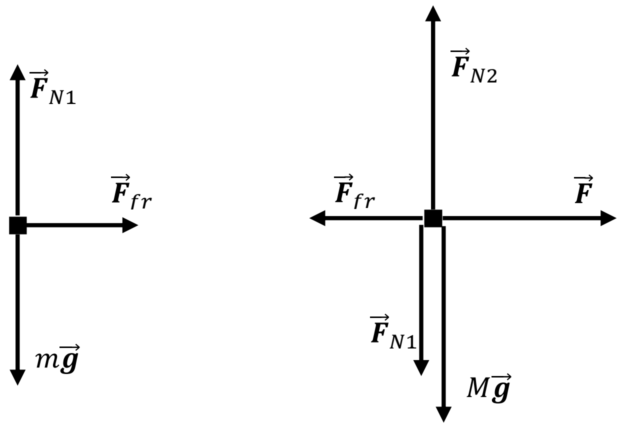
(a) (15 Pts.) Find F_{min} , the constant force with the least magnitude that must be applied to the board in order to pull the board from under the box (which then will fall off the opposite end of the board).

(b) (15 Pts.) What will be the relative acceleration of the box with respect to the board when it starts to slide with the applied force F_{min}?



Solution:

(a) The free-body diagrams for the two masses are given in the figure. Since the box does not slide, both the box and the board will have the same horizontal acceleration *a*. Applying Newton's second law to the first figure, we have



$F_{fr} = m\,a$ in the horizontal direction, and

$F_{N1} - m\,g = 0$ in the vertical direction. We also know that $F_{fr} \le \mu_s F_{N1}$. Therefore,

$m\,a \le \mu_s\, m\,g \;\rightarrow\; a \le \mu_s\, g$.

Since both the box and the board move with the same acceleration *a*, as a single object with mass $m+M$, we can write

$F = (m+M)a \;\rightarrow\; a = \dfrac{F}{(m+M)}$. Hence we must have $\dfrac{F}{(m+M)} \le \mu_s\, g \;\rightarrow\; F \le \mu_s\,(m+M)\,g$ for the box not to slide on the board. This means that the constant force with the least magnitude needed to make the box slide is $F_{\min} = \mu_s\,(m+M)\,g$.

(b) İf the box starts to slide with the applied force $F_{\min}$, the effective friction force is due to kinetic friction, $F_{fr} = \mu_k F_{N1} = \mu_k mg$. In this case, applying Newton's second law to the free body diagrams, we have

$F_{fr} = m\,a_{XS} \;\rightarrow\; \mu_k mg = ma_{XS}$. So, the acceleration of the box with respect to the surface is $a_{XS} = \mu_k g$.

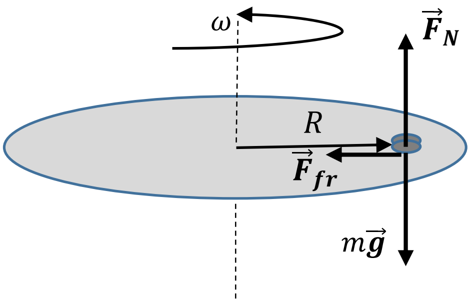
$F_{\min} - F_{fr} = M\,a_{BS} \;\rightarrow\; \mu_s\,(m+M)\,g - \mu_k mg = Ma_{BS}$ means acceleration of the board with respect to the surface is $a_{BS} = \dfrac{1}{M}\left[(m+M)\mu_s - m\mu_k\right]g$.

Since $a_{XS} = a_{XB} + a_{BS}$, relative acceleration a_{XB} of the box with respect to the board will be

$a_{XB} = a_{XS} - a_{BS} = \mu_k g - \dfrac{1}{M}\left[(m+M)\mu_s - m\mu_k\right]g$, or

$a_{XB} = \left(1 + \dfrac{m}{M}\right)\left(\mu_k - \mu_s\right)g$. Since $\mu_k < \mu_s$, a_{XB} is negative.

4. (20 Pts.) A coin is placed a distance R from the axis of a rotating turntable of variable speed. The coefficient of static friction between the coin and the turntable is μ_s. If the angular speed ω of the turntable is slowly increased, at which angular speed does the coin slide off?



Solution:

The centripetal force responsible for the circular motion of the coin is the force of static friction between the coin and the turntable. Applying Newton's second law to the free-body diagram in the figure, we have

$F_{fr} = m a_c = m R \omega^2$ in the radial direction, and $F_N - m g = 0$ in the vertical direction. We also know that $F_{fr} \le \mu_s F_N$ for the case of static friction. Putting these together, we find

$$F_{fr} = m R \omega^2 \le \mu_s m g \;\rightarrow\; \omega^2 \le \frac{\mu_s g}{R}.$$

Therefore, for the coin to slide off we have

$$\omega_{\min} = \sqrt{\frac{\mu_s g}{R}}.$$