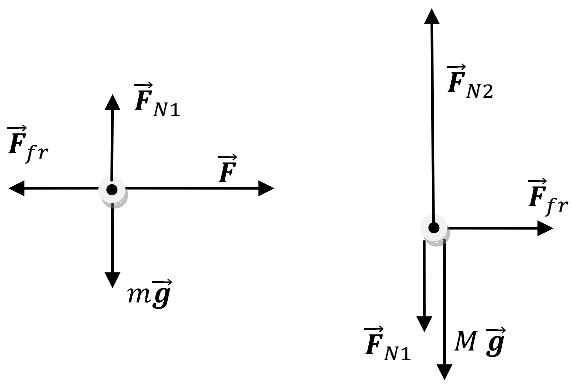
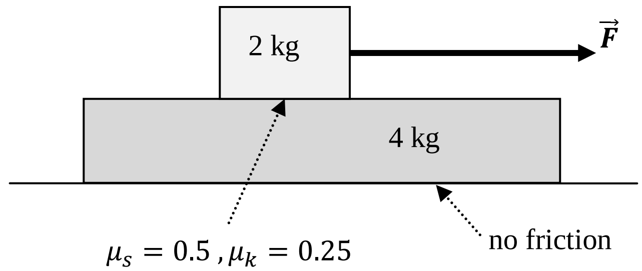
PHYS 101 – General Physics I, Midterm Exam 2 Solutions

1. A 2 *kg* block is stacked on top of a 4 *kg* block which is free to slide on a frictionless horizontal surface. The coefficient of static friction between the two blocks is 0.5, while the coefficient of kinetic friction is 0.25. A horizontal force $\vec{F}$ is applied to the top block as shown in the figure.

(a)(4 Pts.) Draw a free-body diagram for each block.





(b)(8 Pts.) Taking $g = 10\,m/s^2$, determine the maximum force F that will cause the two blocks to move as one (i.e., the top block will not slide on the bottom one).

$F_{max} = 15$ N

Let *m* and *M* denote masses of the top and the bottom blocks respectively. Since both blocks move as one, they have the same acceleration $\vec{a}$, $a_x = a\,, a_y = 0$. Writing Newton's second law for the top block, we have

$$F - F_{fr} = ma\,,\ \ F_{N1} - mg = 0$$

Since the two blocks dont move relative to each other, the effective friction is static. So we have

$F_{fr} \leq \mu_s F_{N1}$. Using the second equation above, we get $F_{fr} \leq \mu_s mg$.

Writing Newton's second law for the bottom block, we have $F_{fr} = Ma$. So $F - Ma = ma$, or $a = \dfrac{F}{M+m}$.

Using this result in the first equation, we get $F_{fr} = F - m\left(\dfrac{F}{M+m}\right) = \dfrac{M\,F}{M+m} \leq \mu_s mg$, or

$F \leq \mu_s \dfrac{m}{M}\left(M+m\right)g$. So

$$F_{max} = \mu_s \frac{m}{M}\left(M+m\right)g = \left(\frac{1}{2}\right)\left(\frac{2kg}{4kg}\right)\left(6kg\right)\left(10\,{}^{m}\!/\!{}_{s^2}\right) = 15\,N\,.$$

(c) (8 Pts.) If F = 25 N, what will be the acceleration of each block?

$a_{top} = 10\ {}^{m}/_{s^2}$

$a_{bottom} = \frac{5}{4}\ {}^{m}/_{s^2}$

In this case the accelerations of the two blocks will not be the same and, since the top block will be sliding on the bottom one, effective friction is kinetic. Newton's second law for the top block is written as
$F - F_{fr} = ma_{top} \rightarrow ma_{top} = F - \mu_k mg$, or

$$(2\,kg)a = 25N - \frac{1}{4}(2\,kg)(10\,m/s^2) \rightarrow a = 10\,m/s^2\ .$$

Newton's second law for the bottom block gives

$$F_{fr} = Ma_{bot} \ \rightarrow\ a_{bot} = \frac{F_{fr}}{M} = \frac{\mu_k mg}{M} = \frac{1}{4}\frac{2}{4}10\,m/s^2 = \frac{5}{4}m/s^2\,.$$

2. Two stars with equal mass *M*, whose centers are a distance *d* apart, rotate about the point midway between them with a period *T*.

(a)(5 Pts.) What is the magnitude of the force exerted by one star onto the other?

$F_{12} = GM^2/d^2$

(b)(7 Pts.) Find the expression for their mass *M* in terms of *G*, *T* and *d*.

$M = \dfrac{2\pi^2 d^3}{GT^2}$

Each star rotates around the circle with radius $d/2$, so we have

$$\frac{GM^2}{d^2} = Ma_c = M\frac{v^2}{(d/2)} \ \rightarrow\ M = \frac{2dv^2}{G}.$$

Since the period is *T*, we have

$$vT = 2\pi\frac{d}{2} = \pi d \ \rightarrow\ v = \frac{\pi d}{T} \ \rightarrow\ M = \frac{2\pi^2 d^3}{GT^2}.$$

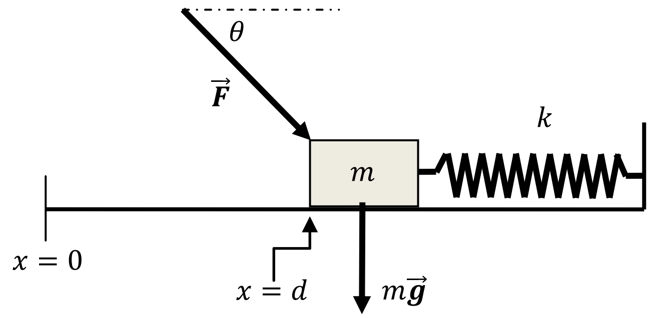
(c)(7 Pts.) Find the expression for the total mechanical energy of the two stars in terms of *G*, *T* and *d*.

$E = -\dfrac{2\pi^4 d^5}{GT^4}$

$E = K_1 + K_2 + U = \frac{1}{2}Mv^2 + \frac{1}{2}Mv^2 - G\frac{M^2}{d} = Mv^2 - G\frac{M^2}{d}$. Using results of the previous part, we have

$$E = Mv^2 - G\frac{M^2}{d} = \left(\frac{2\pi^2 d^3}{GT^2}\right)\left(\frac{\pi d}{T}\right)^2 - \frac{G}{d}\left(\frac{2\pi^2 d^3}{GT^2}\right)^2 \ \rightarrow\ E = -\frac{2\pi^4 d^5}{GT^4}.$$

3. A block of mass m is on a frictionless horizontal surface and is attached to a spring whose other end is fixed. The stiffness constant of the spring is k . A constant force of magnitude F acts on the block as shown in the figure, holding the spring compressed d meters from its equilibrium position. Initially the block is at rest.



(a)(5 Pts.) Find the expression for the magnitude of the force $\vec{F}$ in terms of the other parameters.

Initially the system is at rest, so the net force on the block is zero. This means

$$F\cos\theta - k\,d = 0,\; F_N - F\sin\theta - mg = 0.$$

$$F = \frac{k\,d}{\cos\theta}$$

(b)(5 Pts.) Suddenly the force $\vec{F}$ is removed and the block starts to move. What is the work done by the force of gravity while the block moves from $x = d$ to $x = d/2$?

$$W_g = 0$$

The force of gravity is perpendicular to the displacement vector.

(c)(5 Pts.) What is the work done by the spring force while the block moves from $x = d$ to $x = d/2$?

$$W_s = \int_d^{d/2} (-kx\hat{i})\cdot(dx\hat{i}) = -k\int_d^{d/2} x\,dx = -\frac{1}{2}k\left(\frac{d}{2}\right)^2 + \frac{1}{2}k\left(d\right)^2 = \frac{3}{8}kd^2.$$

$$W_s = \frac{3}{8}kd^2$$

(d)(5 Pts.) What will be the velocity of the block at $x = d/2$?

$$v = \frac{d}{2}\sqrt{\frac{3k}{m}}$$

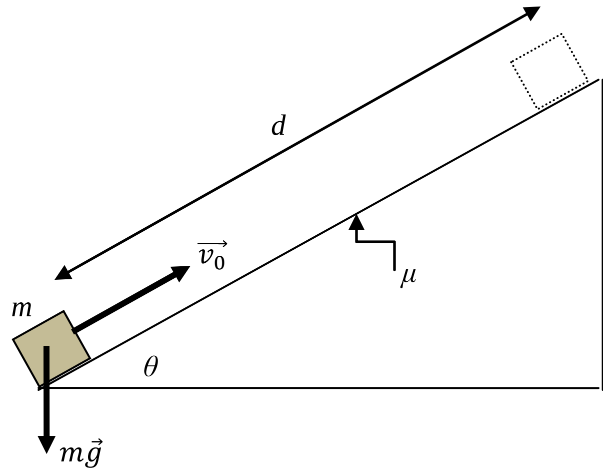
Using the work-energy principle, we have

$$W_{net} = \Delta K = K_f - K_i \;\rightarrow\; W_{net} = \frac{1}{2}m v_f^2 - 0.$$

Since only the spring force does work, we have

$$v_f^2 = \frac{2W_{net}}{m} \;\rightarrow\; v_f = \sqrt{\frac{3kd^2}{4m}} = \frac{d}{2}\sqrt{\frac{3k}{m}}.$$

4. A block of mass $m = 2\,kg$ is given an initial velocity $\vec{v}_0$ up an inclined plane with angle of inclination θ. The coefficient of kinetic friction between the block and the inclined plane is $\mu = 0.5$. It is observed that the block travels a distance $d = 5\,m$ up the inclined plane before coming to rest. Take $g = 10\,m/s^2$, $\sin\theta = 3/5$.



(a)(10 Pts.) Use the **work energy theorem** to find the initial speed v_0.

$v_0 = 10\,m/s^2$

$F_{fr} = \mu_k mg\cos\theta \rightarrow W_{fr} = -\mu_k mgd\cos\theta$.

$W_g = -mgd\sin\theta$, $W_{F_N} = 0$. Work energy theorem: $W_{net} = \Delta K = K_f - K_i$. So, we have

$0 - \frac{1}{2}mv_0^2 = -mgd(\sin\theta + \mu_k\cos\theta) \rightarrow v_0 = \sqrt{2gd(\sin\theta + \mu_k\cos\theta)}$. Using the numerical values given, we find $v_0 = \sqrt{2(10m/s^2)(5m)\left(\frac{3}{5} + \frac{1}{2}\frac{4}{5}\right)} = 10\,m/s$

(b)(10 Pts.) Find the power dissipated by the friction force.

$P_f = -40W$

Chosing the direction of the initial velocity along the inclined plane as the *x*-direction and using Newton's second law, we have

$$a_x = \frac{F_{net}}{m} = -g(\sin\theta + \mu_k\cos\theta) = -(10\,m/s^2)\left(\frac{3}{5} + \frac{1}{2}\frac{4}{5}\right) = -10\,m/s^2.$$

So the time it takes for the block to reach the top is

$$v^2 - v_0^2 = 2ad \;\rightarrow\; 0 - 100(m/s)^2 = -2(10\,m/s^2)(5m)t \;\rightarrow\; t = 1s.$$

Since, for a constant friction force $P_{fr} = \frac{W_{fr}}{t}$, we find

$$P_{fr} = -\mu_k mgd\cos\theta / t = -\frac{1}{2}(2\,kg)(10\,m/s^2)(5\,m)\left(\frac{4}{5}\right)/(1s) = -40W.$$

5. Two blocks are on a frictionless horizontal tabletop. Block A has mass *m* and slides at a speed *v* toward block B of mass 4*m*, which is initially at rest. A coil spring, which obeys Hooke's law and has spring constant *k*, is attached to block B in such a way that it will be compressed when struck by block A, as shown in the figure below. (Ignore the mass of the spring.)

(a)(6 Pts.) What will be the velocities of the two blocks at the instant of maximum compression?

At the instant of maximum compression velocities of the two blocks will be the same. So we let $v_A = v_B = v'$ at that instant and choose the initial direction of block A as the positive direction. Since momentum is conserved, we have

$v_A = v/5$

$v_B = v/5$

$$p_i = p_f \;\rightarrow\; mv = mv' + 4mv' = 5mv' \;\rightarrow\; v' = v/5.$$

(b)(7 Pts.) What will be the maximum compression of the spring?

Since total mechanical energy is also conserved, at the instant of maximum compression we have

$x_{\max} = 2v\sqrt{\dfrac{m}{5k}}$

$$E_i = E_f \;\rightarrow\; \frac{1}{2}mv^2 = \frac{1}{2}(m+4m)v'^2 + \frac{1}{2}k\,x_{\max}^2 .$$

Using the result of part (a), we find

$$mv^2 = (5m)\left(\frac{v}{5}\right)^2 + k\,x_{\max}^2 \;\rightarrow\; x_{\max} = \sqrt{\frac{4mv^2}{5k}} = 2v\sqrt{\frac{m}{5k}}.$$

(*c*)(7 Pts.) It is observed that long after the collision, when the two blocks are separated, the velocity of block A is $\acute{v}_A = -3v/5$. What is the magnitude and the direction of the impulse delivered to block A by block B?

$I = -\dfrac{8}{5}mv$

$$I = \Delta p = p_f - p_i \;\rightarrow\; I = -\frac{3}{5}mv - mv = -\frac{8}{5}mv.$$