PHYS 101 – General Physics I Midterm Exam 1 Solution

1. A particle is moving at constant acceleration along the *x*-axis. At time $t=1s$, it is observed to be at the origin ($x=0$) and not moving ($v=0$). Later, at time $t=2s$, its velocity is mesured as 2 *m/s*. (a) Find the equation describing the motion of the object (i.e., find $x(t)$). (b) What is the displacement of the object in the time interval $t_1=0$ and $t_2=2s$? (c) What is the total distance covered by the object in the time interval $t_1=0$ and $t_2=2s$?

Solution: (a) Motion at constant acceleration means

$$x(t)=x_0+v_0t+\frac{1}{2}at^2,\ v(t)=v_0+at.$$

At time $t=1s$, we have

$$x(1)=x_0+v_0+\frac{1}{2}a=0 \quad \text{and} \quad v(1)=v_0+a=0.$$

At time $t=2s$, we have $v(2)=v_0+2a=2$. Solving these three equations, we get

$$x_0=1m, v_0=-2m/s \quad \text{and} \quad a=2m/s^2.$$

Therefore, the equation describing the motion of the object is

$$x(t)=(1-2t+t^2)m.$$

(b) At time t = 0 the object is at $x(0)$ = 1 *m*. At time t = 1 *s*, the object is at $x(1)$ = 0, and at time t = 2 *s* the object is at $x(2)$ = 1 *m*. Therefore, its displacement is $\underline{x(2)-x(0)=0}$. In between, the object is at the origin since $x(1)$ = 0. Hence, the total distance traveled is

$$|x(2)-x(1)|+|x(1)-x(0)|=2m.$$

2. At time $t=0$ a package is dropped from an airplane flying at an altitude of 500 *m* along a straight course at a constant speed of 100 *m/s*. Five seconds later, another package is dropped. Neglect air resistance and take $g=10m/s^2$. Let the horizontal direction in which the plane is flying be the increasing *x*-direction and the increasing *y*-direction be downward towards the ground. (a) When does the first package hit the ground? (b) What is the velocity vector of the first package just before it hits the ground? (c) What is the position vector of the second package at the instant the first one hits the ground? (d) How far away are the packages when both hit the ground?

Solution: (a) For the first package

$$x_1(t) = v_{1x0}t\,,\ y_1(t) = \frac{1}{2}gt^2\,,\ v_{1y}(t) = gt\,.$$

The first package hits the ground means

$$y_1(t) = \frac{1}{2}gt^2 = 5t^2 = 500\,.$$

So, the first package hits the ground 10 *s* after it is dropped.

(b) At *t* = 10 *s*, we have

$$v_{1x} = v_{1x0} = 100\,m/s\,,\ v_{1y}(t = 10\,s) = gt = 100\,m/s\,,$$

so, when the first package hits the ground its velocity vector is

$$\vec{v}_1(t = 10\,s) = (100\,\hat{i} + 100\,\hat{j})\,m/s\,.$$

(c) For the second package we have

$$x_2(t) = x_{20} + v_{2x0}(t-5),\ y_2(t) = \tfrac{1}{2}g(t-5)^2\,,\ v_{2y}(t) = g(t-5)\,.$$
$$x_2(t) = (500\,m) + (100\,m/s)(t-5),\ y_2(t) = \tfrac{1}{2}g(t-5)^2\,,\ v_{2y}(t) = g(t-5)$$

Since the first package hits the ground at time *t* = 10 *s*, and

$$x_2(t = 10\,s) = 1000\,m,\ y_2(t = 10\,s) = 125\,m\,,$$

the position vector of the second package at the instant the first one hits the ground is $\vec{r}_2(t = 10\,s) = (1000\,\hat{i} + 125\,\hat{j})\,m$. (d) The second package hits the ground when

$$y_2(t) = 5(t-5)^2 = 500 \Rightarrow t = 15\,s\,.$$

Since

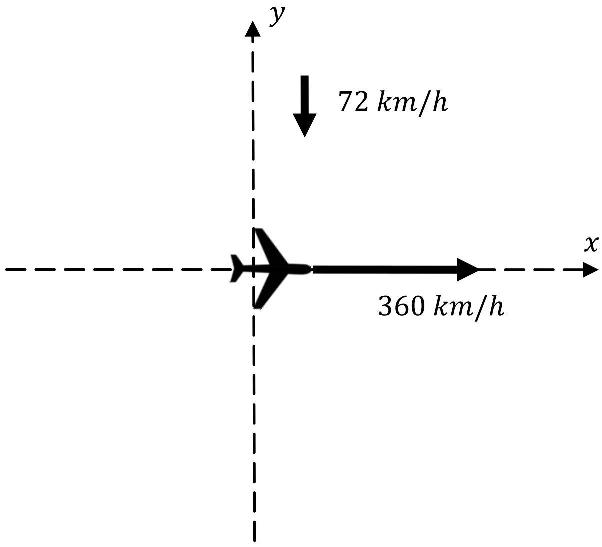
$$x_2(t = 15\,s) = (100\,m/s)(15-5)(s) = 1000\,m\,,$$

and

$$x_2(t = 15\,s) - x_1(t = 10\,s) = 1500\,m - 1000\,m = 500\,m\,,$$

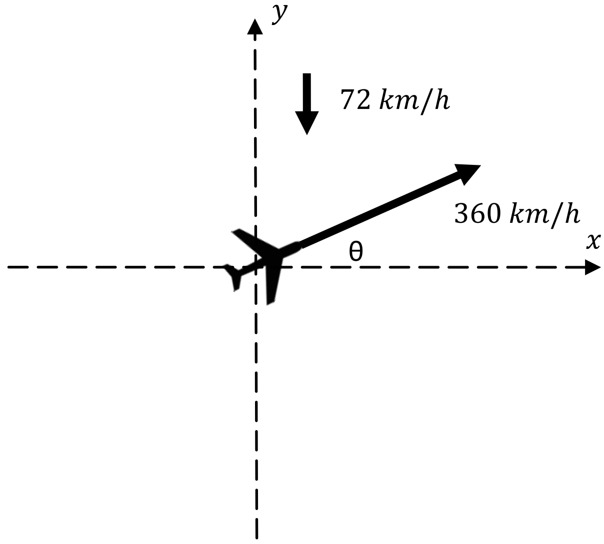
the packages are 500 *m* away when both hit the ground.

3. An airplane is heading due east at a speed of 360 km/h. A wind begins blowing from the north at a speed of 72 km/h (average). (a) Let east be the positive *x*-direction, north be the positive *y*-direction, and calculate the velocity vector of the plane relative to the ground in terms of the unit vectors. (b) In what direction should the pilot aim the plane if it wants to fly due east without changing its speed relative to air (i.e., what is the sin of the angle the plane's velocity vector makes with the x-axis)?



Solution: (a) We have $\vec{v}_{PG} = \vec{v}_{PA} + \vec{v}_{AG}$, where $\vec{v}_{PA} = (100\,m/s)\,\hat{i}$ is the velocity of the airplane with respect to the air, $\vec{v}_{AG} = (-20\,m/s)\,\hat{j}$ is the velocity of the air (wind) with respect to the ground. Hence, the velocity vector of the plane relative to the ground is

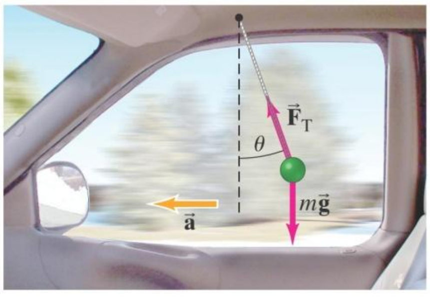
$\vec{v}_{PG} = (100\,m/s)\,\hat{i} - (20\,m/s)\,\hat{j}$.



(b) If the plane is to fly due east it must aim slightly into the wind. The sin of the angle is

$$\sin\theta = \frac{v_{AG}}{v_{PA}} = \frac{20\,m/s}{100\,m/s} = 0.2 \ .$$

4. (Example 4-15 on page 100 of the textbook.) A small mass m hangs from a thin string and can swing like a pendulum. It is attached above the window of a car as shown in the figure on the left. When the car is at rest, the string hangs vertically. What angle θ does the string make (a) when the car accelerates at a constant $a = 2.0\,m/s^2$, and (b) when the car moves at constant velocity $v = 90\,km/h$? (take $g = 10\,m/s^2$)



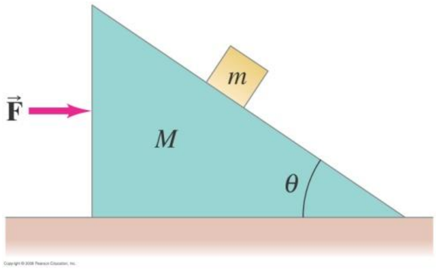
(b)

Solution: The acceleration $a = 2.0\,m/s^2$ is horizontal. So, choosing the direction of the acceleration as the positive x-direction and letting the positive y-direction be upward, we have $a_x = 2.0\,m/s^2$, $a_y = 0$. From Newton's second law, we have $F_T \sin\theta = ma_x$ in the x-direction and $F_T \cos\theta - mg = 0$ in the y-direction. From these two equations, we get

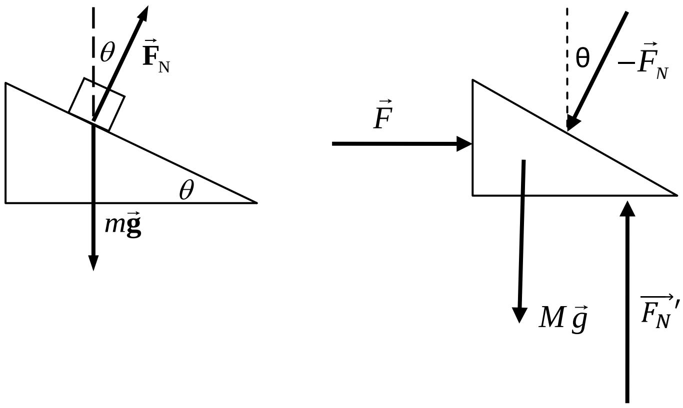
$$\tan\theta = \frac{a_x}{g} = \frac{2.0}{10} = 0.2 .$$

(b) When the car moves at constant velocity we have $a_x = 0$, and hence $\tan\theta = \frac{a_x}{g} = 0 \Rightarrow \theta = 0$.

5. (Problem 55 on page 108 of the textbook.) A small block of mass m rests on the sloping side of a triangular block of mass M which itself rests on a horizontal table as shown in the figure on the left. Assume all surfaces are frictionless and take x and y axes horizontal and vertical. (a) Draw a free-body diagram for each object. (b) In terms of the given parameters m, M, θ and g, determine the magnitude of the force $\vec{\mathbf{F}}$ that must be applied to M so that m remains in a fixed position relative to M (that is, m doesn't move on the incline).



Solution: (a)



(b) If m doesn't move on the incline, it doesn't move in the vertical direction, and so has no vertical component of acceleration. This suggests that we analyze the forces parallel and perpendicular to the floor. See the force diagram for the small block, and use Newton's second law to find the acceleration of the small block.

$$\sum F_y = F_N \cos\theta - mg = 0 \quad \rightarrow \quad F_N = \frac{mg}{\cos\theta}$$

$$\sum F_x = F_N \sin\theta = ma \quad \rightarrow \quad a = \frac{F_N \sin\theta}{m} = \frac{mg\sin\theta}{m\cos\theta} = g\tan\theta$$

Since the small block doesn't move on the incline, the combination of both masses has the same horizontal acceleration of $g\tan\theta$. That can be used to find the applied force.

$$F_{\text{applied}} = (m+M)a = \boxed{(m+M)g\tan\theta}$$