3a. Suppose that the radius of convergence of the power series $\sum a_n x^n$ is R_1, the radius of convergence of the power series $\sum b_n x^n$ is R_2, and the radius of convergence of the power series $\sum (a_n + b_n)x^n$ is R. Prove that if $R_1 < R_2$, then $R = R_1$.

If $|x| < R_1$, then both $\sum a_n x^n$ and $\sum b_n x^n$ converges; hence their sum $\sum (a_n + b_n) x^n$ converges. $\Rightarrow R \geq R_1$.

If $R_1 < |x| < R_2$, then $\sum a_n x^n$ converges and $\sum b_n x^n$ diverges; hence their sum $\sum (a_n + b_n) x^n$ diverges $\Rightarrow R \leq R_1$.

Therefore, $R = R_1$.

3b. Fill in the boxes. No explanation is required.

The radius of convergence of the power series $\sum_{n=0}^{\infty} 2^n x^n$ is $\boxed{\frac{1}{2}}$.

The radius of convergence of the power series $\sum_{n=0}^{\infty} 3^n x^n$ is $\boxed{\frac{1}{3}}$.

The radius of convergence of the power series $\sum_{n=0}^{\infty} (2^n + 3^n) x^n$ is $\boxed{\frac{1}{3}}$.

3c. Find all solutions of the equation $\sum_{n=0}^{\infty} (2^n + 3^n) x^n = 6$.

$\sum_{n=0}^{\infty} 2^n x^n = \frac{1}{1-2x}$ for $|x| < \frac{1}{2}$ and $\sum_{n=0}^{\infty} 3^n x^n = \frac{1}{1-3x}$ for $|x| < \frac{1}{3}$

$\Rightarrow \sum_{n=0}^{\infty} (2^n + 3^n) x^n = \frac{1}{1-2x} + \frac{1}{1-3x} = \frac{2-5x}{6x^2-5x+1}$ for $|x| < \frac{1}{3}$.

$\sum_{n=0}^{\infty} (2^n + 3^n) x^n = 6 \Rightarrow \frac{2-5x}{6x^2-5x+1} = 6 \Rightarrow 36x^2 - 25x + 4 = 0 \Rightarrow (4x-1)\cdot(9x-4)$

$\Rightarrow \begin{cases} x = \frac{1}{4} \\ \text{or} \\ x = \frac{4}{9} \end{cases}$

Since $|x| = \left|\frac{4}{9}\right| = \frac{4}{9} > \frac{1}{3}$, the series diverges at $x = \frac{4}{9}$.

The only solution is $x = \frac{1}{4}$.