3. Determine the interval of convergence of the power series:

$$\sum_{n=0}^{\infty} \frac{2^n}{5^n + (-3)^n} x^n$$

$$a_n = \frac{2^n}{5^n + (-3)^n} x^n = \frac{1}{1+\left(-\frac{3}{5}\right)^n} \cdot \frac{2^n x^n}{5^n}$$

$$L = \lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = \lim_{n\to\infty} \frac{\left|\frac{1}{1+\left(-\frac{3}{5}\right)^{n+1}} \cdot \frac{2^{n+1}x^{n+1}}{5^{n+1}}\right|}{\left|\frac{1}{1+\left(-\frac{3}{5}\right)^n} \cdot \frac{2^n x^n}{5^n}\right|} = \frac{2|x|}{5} \cdot \lim_{n\to\infty} \frac{1+\overbrace{\left(-\frac{3}{5}\right)^n}^{\to 0}}{1+\underbrace{\left(-\frac{3}{5}\right)^{n+1}}_{\to 0}} = \frac{2|x|}{5}$$

⊛ If $|x| < \frac{5}{2}$, then $L < 1$ and the power series converges.

⊛ If $|x| > \frac{5}{2}$, then $L > 1$ and the power series diverges.

⊛ $x = \frac{5}{2} \Rightarrow \sum_{n=0}^{\infty} \frac{2^n}{5^n + (-3)^n} x^n = \sum_{n=0}^{\infty} \frac{2^n}{5^n + (-3)^n} \cdot \left(\frac{5}{2}\right)^n = \sum_{n=0}^{\infty} \frac{1}{1+\left(-\frac{3}{5}\right)^n}$

Since $\lim_{n\to\infty} \frac{1}{1+\left(-\frac{3}{5}\right)^n} = 1 \neq 0$, the power series diverges when $x = \frac{5}{2}$ by n^{th} Term Test.

⊛ $x = -\frac{5}{2} \Rightarrow \sum_{n=0}^{\infty} \frac{2^n}{5^n + (-3)^n} x^n = \sum_{n=0}^{\infty} \frac{2^n}{5^n + (-3)^n} \cdot \left(-\frac{5}{2}\right)^n = \sum_{n=0}^{\infty} (-1)^n \cdot \frac{1}{1+\left(-\frac{3}{5}\right)^n}$

Since $\lim_{n\to\infty} \frac{1}{1+\left(-\frac{3}{5}\right)^n} = 1 \neq 0$, $\lim_{n\to\infty}\left((-1)^n \cdot \frac{1}{1+\left(-\frac{3}{5}\right)^n}\right)$ does not exist.

Hence the power series diverges when $x = -\frac{5}{2}$ by n^{th} Term Test.

The interval of convergence of the power series is $\left(-\frac{5}{2}, \frac{5}{2}\right)$.