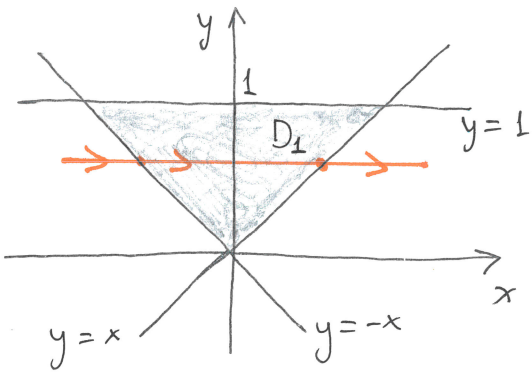
4a. Evaluate double integral $I_1 = \iint_{D_1} x^2 y \cos(\pi y^5/2)\, dA$ where $D_1 = \{(x,y) : -y \le x \le y \text{ and } y \le 1\}$.



$$I_1 = \int_0^1 \int_{-y}^{y} x^2 y \cos\left(\tfrac{\pi}{2} y^5\right) dx\, dy$$

$$= \int_0^1 \left[\tfrac{1}{3} x^3 y \cos\left(\tfrac{\pi}{2} y^5\right)\right]_{x=-y}^{x=y} dy$$

$$= \int_0^1 \left(\tfrac{1}{3} y^3 y \cos\left(\tfrac{\pi}{2} y^5\right) - \tfrac{1}{3} (-y)^3 y \cos\left(\tfrac{\pi}{2} y^5\right)\right) dy$$

$$= \int_0^1 \tfrac{2}{3} y^4 \cos\left(\tfrac{\pi}{2} y^5\right) dy = \tfrac{2}{3} \cdot \tfrac{2}{5\pi} \sin\left(\tfrac{\pi}{2} y^5\right)\Big]_0^1 = \tfrac{4}{15\pi} \sin\left(\tfrac{\pi}{2}\right) = \tfrac{4}{15\pi}$$

4b. Express the double integral $I_2 = \iint_{D_2} f(x,y)\, dA$ in terms of iterated integrals in polar coordinates where $D_2 = \{(x,y) : x^2 + y^2 \le 2 \text{ and } y + \sqrt{3}x \ge 0\}$.

$$I_2 = \int_{-\frac{\pi}{3}}^{\frac{2\pi}{3}} \int_0^{\sqrt{2}} f(r\cos\theta, r\sin\theta)\, r\, dr\, d\theta$$