3. Determine whether each of the following series converges or diverges.

a. $\sum_{n=1}^{\infty}\left(\frac{1}{\sqrt{n}}-\frac{1}{\sqrt{n+1}}\right)^2$

$$0<\frac{1}{\sqrt{n+1}}<\frac{1}{\sqrt{n}}\le 1 \Rightarrow 0<\frac{1}{\sqrt{n}}-\frac{1}{\sqrt{n+1}}<1 \text{ for } n\ge 1$$

$$\left.\begin{array}{l}\Rightarrow 0<\left(\frac{1}{\sqrt{n}}-\frac{1}{\sqrt{n+1}}\right)^2<\frac{1}{\sqrt{n}}-\frac{1}{\sqrt{n+1}} \text{ for } n\ge 1 \\ \sum_{n=1}^{\infty}\left(\frac{1}{\sqrt{n}}-\frac{1}{\sqrt{n+1}}\right) \text{ converges by } \underline{Q2} \text{ } \underline{\text{Part a}}\end{array}\right\} \Rightarrow \sum_{n=1}^{\infty}\left(\frac{1}{\sqrt{n}}-\frac{1}{\sqrt{n+1}}\right)^2 \text{ converges by DCT.}$$

b. $\sum_{n=1}^{\infty}\sqrt{\frac{1}{\sqrt{n}}-\frac{1}{\sqrt{n+1}}}$

$$\frac{1}{\sqrt{n}}-\frac{1}{\sqrt{n+1}}=\frac{\sqrt{n+1}-\sqrt{n}}{\sqrt{n}\cdot\sqrt{n+1}}=\frac{1}{\sqrt{n}\cdot\sqrt{n+1}\cdot(\sqrt{n+1}+\sqrt{n})}$$

$$c=\lim_{n\to\infty}\frac{\sqrt{\frac{1}{\sqrt{n}}-\frac{1}{\sqrt{n+1}}}}{\frac{1}{n^{3/4}}}=\left(\lim_{n\to\infty}\frac{1}{\sqrt{1+\frac{1}{n}}\cdot\left(\sqrt{1+\frac{1}{n}}+1\right)}\right)^{1/2}=\left(\frac{1}{\sqrt{1+0}\cdot(\sqrt{1+0}+1)}\right)^{1/2}=\frac{1}{\sqrt{2}}$$

Since $c=\frac{1}{\sqrt{2}}>0$ and $\sum_{n=1}^{\infty}\frac{1}{n^{3/4}}$ diverges (p-series with $p=\frac{3}{4}\le 1$),

$\sum_{n=1}^{\infty}\sqrt{\frac{1}{\sqrt{n}}-\frac{1}{\sqrt{n+1}}}$ diverges by LCT.