

2. Determine whether each of the following series converges or diverges.

a. $\sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}} \right)$

$$s_n = \left(\frac{1}{\sqrt{1}} - \frac{1}{\sqrt{2}} \right) + \left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}} \right) + \dots + \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}} \right) = 1 - \frac{1}{\sqrt{n+1}} \quad \text{for } n \geq 1$$

$$\Rightarrow \lim_{n \rightarrow \infty} s_n = \lim_{n \rightarrow \infty} \left(1 - \frac{1}{\sqrt{n+1}} \right) = 1 - 0 = 1$$

$$\Rightarrow \sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}} \right) = 1 \Rightarrow \sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}} \right) \text{ converges.}$$

b. $\sum_{n=1}^{\infty} (-1)^{n-1} \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}} \right)$

$$\sum_{n=1}^{\infty} \left| (-1)^{n-1} \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}} \right) \right| = \sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}} \right) \text{ converges by } \underline{\text{Part a}}.$$

$$\Rightarrow \sum_{n=1}^{\infty} (-1)^{n-1} \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}} \right) \text{ converges by } \underline{\text{ACT}}.$$