1. Let $a_1 = 1$ and $a_n = 1 + \dfrac{8}{1 + a_{n-1}}$ for $n \geq 2$, and let $b_n = \dfrac{2^n - (-1)^n}{3}$ for $n \geq 1$.

a. Fill in the following boxes.

$a_2 = \boxed{5}$ $\quad a_3 = \boxed{\frac{7}{3}}$ $\quad a_4 = \boxed{\frac{17}{5}}$ $\quad a_5 = \boxed{\frac{31}{11}}$

b. Fill in the following boxes.

$b_2 = \boxed{1}$ $\quad b_3 = \boxed{3}$ $\quad b_4 = \boxed{5}$ $\quad b_5 = \boxed{11}$

c. Guess an explicit formula for a_n.

$a_n = \boxed{3 \cdot \dfrac{2^n + (-1)^n}{2^n - (-1)^n}}$ for $n \geq 1$

d. Prove that your guess in **Part c** is correct.

Proof by induction:

* $n = 1 \Rightarrow 3 \cdot \dfrac{2^n + (-1)^n}{2^n - (-1)^n} = 3 \cdot \dfrac{2 + (-1)}{2 - (-1)} = 3 \cdot \dfrac{1}{3} = 1 = a_1$

* Suppose $a_k = 3 \cdot \dfrac{2^k + (-1)^k}{2^k - (-1)^k}$ for some $k \geq 1$. Then:

$1 + a_k = 1 + 3 \cdot \dfrac{2^k + (-1)^k}{2^k - (-1)^k} = \dfrac{4 \cdot 2^k + 2 \cdot (-1)^k}{2^k - (-1)^k}$ and

$a_{k+1} = 1 + \dfrac{8}{1 + a_k} = 1 + \dfrac{8 \cdot 2^k - 8 \cdot (-1)^k}{4 \cdot 2^k + 2 \cdot (-1)^k} = \dfrac{12 \cdot 2^k - 6 \cdot (-1)^k}{4 \cdot 2^k + 2 \cdot (-1)^k} = 3 \cdot \dfrac{2^{k+1} + (-1)^{k+1}}{2^{k+1} - (-1)^{k+1}}$

Hence $a_n = 3 \cdot \dfrac{2^n + (-1)^n}{2^n - (-1)^n}$ for all $n \geq 1$

e. Find $\lim\limits_{n \to \infty} a_n$.

$\lim\limits_{n \to \infty} a_n = \lim\limits_{n \to \infty} 3 \cdot \dfrac{2^n + (-1)^n}{2^n - (-1)^n} = 3 \cdot \lim\limits_{n \to \infty} \dfrac{1 + (-1/2)^n}{1 - (-1/2)^n} = 3 \cdot \dfrac{1 + 0}{1 - 0} = 3$