

3. Find the absolute maximum and minimum values of the function $f(x, y) = 2xy - x - 2y^2$ on the square $D = \{(x, y) : 0 \leq x \leq 3 \text{ and } 0 \leq y \leq 3\}$.

Interior of D:

$$\left. \begin{aligned} f_x = 2y - 1 = 0 &\Rightarrow y = \frac{1}{2} \\ f_y = 2x - 4y = 0 &\Rightarrow x = 1 \end{aligned} \right\} \Rightarrow (x, y) = (1, \frac{1}{2})$$

Boundary of D:

Side I: $0 \leq x \leq 3$ and $y = 0$

$$f(x, 0) = -x \text{ for } 0 \leq x \leq 3$$

Critical points: $\frac{d}{dx} f(x, 0) = -1 = 0 \Rightarrow$ no solution

Endpoints: $x = 0, x = 3 \Rightarrow (x, y) = (0, 0), (3, 0)$

Side II: $0 \leq y \leq 3$ and $x = 0$

$$f(0, y) = -2y^2 \text{ for } 0 \leq y \leq 3$$

Critical points: $\frac{d}{dy} f(0, y) = -4y = 0 \Rightarrow y = 0$

Endpoints: $y = 0, y = 3 \Rightarrow (x, y) = (0, 0), (0, 3)$

Side III: $0 \leq x \leq 3$ and $y = 3$

$$f(x, 3) = 5x - 18 \text{ for } 0 \leq x \leq 3$$

Critical points: $\frac{d}{dx} f(x, 3) = 5 = 0 \Rightarrow$ no solution

Endpoints: $x = 0, x = 3 \Rightarrow (x, y) = (0, 3), (3, 3)$

Side IV: $0 \leq y \leq 3$ and $x = 3$

$$f(3, y) = 6y - 3 - 2y^2 \text{ for } 0 \leq y \leq 3$$

Critical points: $\frac{d}{dy} f(3, y) = 6 - 4y = 0 \Rightarrow y = \frac{3}{2} \Rightarrow (x, y) = (3, \frac{3}{2})$

Endpoints: $y = 0, y = 3 \Rightarrow (x, y) = (3, 0), (3, 3)$

Absolute maximum value is $\frac{3}{2}$, absolute minimum value is -18 .

