Do not forget to *write* your ***full name*** and your ***Bilkent ID number***, and ***sign*** on the upper right corner of your paper.

Midterm Exam Question 3.

Determine whether each of the following series converges or diverges:

a. $\displaystyle\sum_{n=1}^{\infty}\left(\sqrt{n^2+n}-n\right)$ b. $\displaystyle\sum_{n=1}^{\infty}\left(\sqrt{n^2+n}-n\right)^n$

Show all your work!
Explain your reasoning fully and in detail using correct mathematical notation and terminology, and in well-formed mathematical and English sentences!

(a) $a_n = \sqrt{n^2+n} - n \Rightarrow \lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty}\left(\sqrt{n^2+n}-n\right) = \lim\limits_{n\to\infty} \dfrac{n^2+n-n^2}{\sqrt{n^2+n}+n}$

$= \lim\limits_{n\to\infty} \dfrac{1}{\sqrt{1+\frac{1}{n}}+1} = \dfrac{1}{2} \neq 0 \Rightarrow \displaystyle\sum_{n=1}^{\infty}\left(\sqrt{n^2+n}-n\right)$ diverges by nTT.

(b) $a_n = \left(\sqrt{n^2+n}-n\right)^n$

$L = \lim\limits_{n\to\infty} |a_n|^{1/n} = \lim\limits_{n\to\infty} \left|\left(\sqrt{n^2+n}-n\right)^n\right|^{1/n} = \lim\limits_{n\to\infty}\left(\sqrt{n^2+n}-n\right) = \dfrac{1}{2}$

Since $L = \dfrac{1}{2} < 1$, $\displaystyle\sum_{n=1}^{\infty}\left(\sqrt{n^2+n}-n\right)^n$ converges by nRT.