Do not forget to write your **full name** and your **Bilkent ID number**, and **sign** on the upper right corner of your paper.

Midterm 2 Part 1.

Write your Bilkent student ID number here ☞ ☐☐☐☐☐☐☐☐
A B C D E F G H

You are given the following information about a differentiable function $f(x, y, z)$:

❶ At P_0, f increases in the direction of the vector $\mathbf{A} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$ at a rate of ☐. ☞ *Write* A+G *here*

❷ At P_0, f decreases in the direction of the vector $\mathbf{B} = 2\mathbf{i} + 3\mathbf{j} + 6\mathbf{k}$ at a rate of ☐. ☞ *Write* B+H *here*

Using only this information and knowing nothing else about the function f, find a nonzero vector $\mathbf{C}$ such that the directional derivative of f at P_0 in the direction of $\mathbf{C}$ is 0.

Let $\vec{\nabla} f(P_0) = p\vec{i} + q\vec{j} + r\vec{k}$. Since $D_{\vec{u}} f(P_0) = \vec{\nabla} f(P_0) \cdot \vec{u}$:

(1) $\Downarrow$

$$\vec{u} = \frac{\vec{A}}{|\vec{A}|} = \frac{\vec{i} + 2\vec{j} + 2\vec{k}}{3}$$

$$p + 2q + 2r = 3(A+G) \qquad (\mathrm{I})$$

(2) $\searrow$

$$\vec{v} = \frac{\vec{B}}{|\vec{B}|} = \frac{2\vec{i} + 3\vec{j} + 6\vec{k}}{7}$$

$$2p + 3q + 6r = -7 \cdot (B+H) \qquad (\mathrm{II})$$

Every $\langle p, q, r \rangle$ satisfying (I) and (II) also satisfies:
$7 \cdot (B+H) \times (\mathrm{I}) + 3 \cdot (A+G) \times (\mathrm{II})$:

$$\underbrace{\big(7(B+H) + 6(A+G)\big)}p + \underbrace{\big(14 \cdot (B+H) + 9 \cdot (A+G)\big)}q + \underbrace{\big(14 \cdot (B+H) + 18 \cdot (A+G)\big)}r = 0$$

Therefore we can take:

$$\vec{C} = (\quad)\vec{i} + (\quad)\vec{j} + (\quad)\vec{k}$$