

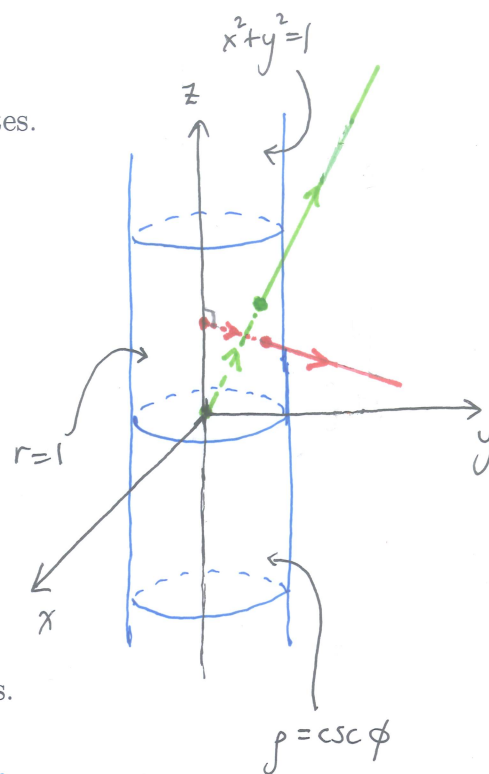
3. Consider the triple integral

$$I = \iiint_E \frac{1}{(x^2 + y^2 + z^2)^2} dV$$

where $E = \{(x, y, z) : x^2 + y^2 \geq 1\}$.

a. Express I in terms of iterated integrals in cylindrical coordinates.

$$I = \int_0^{2\pi} \int_{-\infty}^{\infty} \int_1^{\infty} \frac{1}{(r^2 + z^2)^2} r dr dz d\theta$$



b. Express I in terms of iterated integrals in spherical coordinates.

$$I = \int_0^{2\pi} \int_0^{\pi} \int_{\csc \phi}^{\infty} \frac{1}{(\rho^2)^2} \rho^2 \sin \phi d\rho d\phi d\theta$$

c. Evaluate I .

$$\begin{aligned} I &= \int_0^{2\pi} \int_0^{\pi} \left[-\frac{1}{\rho} \right]_{\rho=\csc \phi}^{\rho=\infty} \sin \phi d\phi d\theta = \int_0^{2\pi} \int_0^{\pi} \sin^2 \phi d\phi d\theta \\ &= \int_0^{2\pi} \int_0^{\pi} \frac{1 - \cos 2\phi}{2} d\phi d\theta = \int_0^{2\pi} \left[\frac{1}{2} \phi - \frac{1}{4} \sin 2\phi \right]_{\phi=0}^{\phi=\pi} d\theta \\ &= \frac{\pi}{2} \int_0^{2\pi} d\theta = \frac{\pi}{2} \cdot 2\pi = \pi^2 \end{aligned}$$