5. Find and classify the critical points of the function $f(x,y) = x^2y + y^2 - cxy$ where c is a constant.

$$\left.\begin{aligned} f_x &= 2xy - cy = 0 \\ f_y &= x^2 + 2y - cx = 0 \end{aligned}\right\} \Rightarrow y\cdot(2x-c) = 0 \Rightarrow y = 0 \quad \text{or} \quad x = \frac{c}{2}$$

$$x^2 - cx = 0 \qquad\qquad \frac{c^2}{4} + 2y - \frac{c^2}{2} = 0$$

$$x = 0 \quad \text{or} \quad x = c \qquad\qquad y = \frac{c^2}{8}$$

$$(x,y) = (0,0),\ (c,0),\ \left(\frac{c}{2}, \frac{c^2}{8}\right)$$

$$\Delta = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix} = \begin{vmatrix} 2y & 2x-c \\ 2x-c & 2 \end{vmatrix}$$

<u>If $c \neq 0$:</u>

$$\Delta(0,0) = \begin{vmatrix} 0 & -c \\ -c & 2 \end{vmatrix} = -c^2 < 0 \Rightarrow (0,0) \text{ is a saddle point}$$

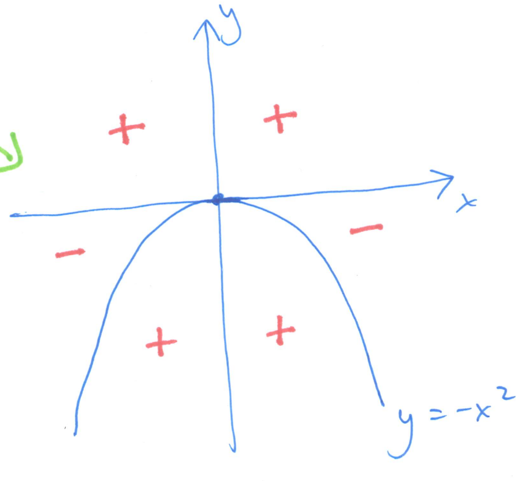
$$\Delta(c,0) = \begin{vmatrix} 0 & c \\ c & 2 \end{vmatrix} = -c^2 < 0 \Rightarrow (c,0) \text{ is a saddle point}$$

$$\Delta\left(\frac{c}{2}, \frac{c^2}{8}\right) = \begin{vmatrix} c^2/4 & 0 \\ 0 & 2 \end{vmatrix} = \frac{c^2}{2} > 0 \text{ and } f_{yy}\left(\frac{c}{2}, \frac{c^2}{8}\right) = 2 > 0$$

$$\Rightarrow \left(\frac{c}{2}, \frac{c^2}{8}\right) \text{ is a local minimum}$$

<u>If $c = 0$:</u>

$$f(x,y) = x^2y + y^2 = (x^2+y)\cdot y$$



(0,0) is a saddle point