**2a.** Make the sentences ❶ and ❷ into true statements by choosing one of the possible completions for each of them. Indicate your choice by marking the ☐ in front of it with a ✓. No explanation is required.

❶ The limit $\lim_{(x,y)\to(0,0)} \frac{x\sin y - y\sin x}{x^2+y^2}$ ☑ exists ☐ does not exist

❷ The limit $\lim_{(x,y)\to(0,0)} \frac{x\sin y - y\sin x}{x^4+y^4}$ ☐ exists ☑ does not exist

**2b.** Choose one of the statements ☐ ❶ and ☑ ❷ you made in part **(2a)**, and prove it. Indicate your choice by marking the ☐ in front of it with a ✓.

$$\lim_{\substack{(x,y)\to(0,0)\\ \text{along the x-axis}}} \frac{x\sin y - y\sin x}{x^4+y^4} = \lim_{x\to 0} \frac{x\cdot\sin 0 - 0\cdot\sin x}{x^4+0^4} = \lim_{x\to 0} 0 = 0$$

$$\lim_{\substack{(x,y)\to(0,0)\\ \text{along the line } y=2x}} \frac{x\sin y - y\sin x}{x^4+y^4} = \lim_{x\to 0} \frac{x\sin 2x - 2x\sin x}{x^4+(2x)^4} = \frac{1}{17}\lim_{x\to 0}\frac{\sin 2x - 2\sin x}{x^3}$$

$$\overset{\text{L'H}}{=} \frac{1}{17}\lim_{x\to 0}\frac{2\cos 2x - 2\cos x}{3x^2} \overset{\text{L'H}}{=} \frac{2}{3\cdot 17}\lim_{x\to 0}\frac{-2\sin 2x + \sin x}{2x}$$

$$= \frac{2}{3\cdot 17}\cdot\left(-2\lim_{x\to 0}\frac{\sin 2x}{2x} + \frac{1}{2}\lim_{x\to 0}\frac{\sin x}{x}\right) = \frac{2}{3\cdot 17}\cdot\left(-2+\frac{1}{2}\right) = -\frac{1}{17}$$

$0 \neq -\frac{1}{17} \Rightarrow \lim_{(x,y)\to(0,0)} \frac{x\sin y - y\sin x}{x^4+y^4}$ does not exist

by 2-Path Test.