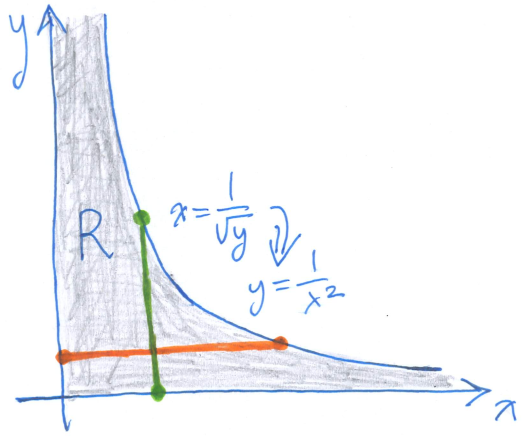
2a. Evaluate the iterated integral $\int_0^\infty \int_0^{1/\sqrt{y}} e^{-1/x}\, dx\, dy$. `[You can use the fact that` $\int_0^\infty e^{-x}\, dx = 1$.`]`

$$\int_0^\infty \int_0^{1/\sqrt{y}} e^{-1/x}\, dx\, dy = \iint_R e^{-1/x}\, dA = \int_0^\infty \int_0^{1/x^2} e^{-1/x}\, dy\, dx$$

$$= \int_0^\infty \left[e^{-1/x} y \right]_{y=0}^{y=1/x^2} dx = \int_0^\infty e^{-1/x} \cdot \frac{1}{x^2}\, dx$$

$$= \int_\infty^0 e^{-u} \cdot (-du) = 1$$

$$\boxed{u = \frac{1}{x} \quad du = -\frac{1}{x^2}\, dx}$$



2b. Evaluate the double integral $\iint_R (x^2 + y^2)\, dA$ where R is the region between the unit circle and the regular hexagon with center at the origin shown in the figure .

$$\iint_R (x^2+y^2)\, dA \overset{\text{by symmetry}}{=} 12 \iint_{R_1} (x^2+y^2)\, dA = 12 \int_0^{\pi/6} \int_1^{\sqrt{3}\sec\theta} r^2 \cdot r\, dr\, d\theta$$

$$= 12 \int_0^{\pi/6} \left[\frac{1}{4} r^4 \right]_{r=1}^{r=\sqrt{3}\sec\theta} d\theta = 3 \int_0^{\pi/6} (9\sec^4\theta - 1)\, d\theta$$

$$= 27 \int_0^{\pi/6} (\tan^2\theta + 1) \cdot \sec^2\theta\, d\theta - 3 \int_0^{\pi/6} d\theta$$

$$= 27 \left[\frac{1}{3}\tan^3\theta + \tan\theta \right]_0^{\pi/6} - 3 \cdot \frac{\pi}{6}$$

$$= 27 \cdot \left(\frac{1}{9\sqrt{3}} + \frac{1}{\sqrt{3}} \right) - \frac{\pi}{2} = 10\sqrt{3} - \frac{\pi}{2}$$