

4. Find the absolute maximum and minimum values of the function $f(x, y) = x^3 - y^2 + x^2y$ on the closed triangular region T shown in the figure below.

Interior of T : $\begin{cases} f_x = 3x^2 + 2xy = 0 \\ f_y = -2y + x^2 = 0 \end{cases} \Rightarrow \begin{cases} 3x^2 + x^3 = 0 \Rightarrow x=0 \text{ or } x=-3 \\ 2y = x^2 \end{cases}$

$y=0$ $y=\frac{9}{2}$

$(x, y) = (0, 0), (-3, \frac{9}{2})$
not in T

Boundary of T :

Side 1: $-2 \leq x \leq 1$ and $y=1$

$$f(x, 1) = x^3 - 1 + x^2 \text{ for } -2 \leq x \leq 1$$

$$\left. \begin{aligned} \frac{d}{dx} f(x, 1) = 3x^2 + 2x = 0 \Rightarrow x=0 \text{ or } x=-\frac{2}{3} \end{aligned} \right\} \Rightarrow (x, y) = (0, 1), \left(-\frac{2}{3}, 1\right), \left(-2, 1\right), (1, 1)$$

Endpoints: $x=-2, x=1$

Side 2: $x=1$ and $-2 \leq y \leq 1$

$$f(1, y) = 1 - y^2 + y \text{ for } -2 \leq y \leq 1$$

$$\left. \begin{aligned} \frac{d}{dy} f(1, y) = -2y + 1 = 0 \Rightarrow y = \frac{1}{2} \end{aligned} \right\} \Rightarrow (x, y) = \left(1, \frac{1}{2}\right), \left(1, -2\right), (1, 1)$$

Endpoints: $y=-2, y=1$

Side 3: $y = -x - 1$ and $-2 \leq x \leq 1$

$$f(x, -x-1) = -2x^2 - 2x - 1 \text{ for } -2 \leq x \leq 1$$

$$\left. \begin{aligned} \frac{d}{dx} f(x, -x-1) = -4x - 2 = 0 \Rightarrow x = -\frac{1}{2} \end{aligned} \right\} \Rightarrow (x, y) = \left(-\frac{1}{2}, -\frac{1}{2}\right), (-2, 1), (1, -2)$$

Endpoints: $x=-2, x=1$

Abs. max is $\frac{5}{4}$

Abs. min is -5

