3. Consider the surfaces $S_1 : xyz = 10$ and $S_2 : z = x^2 + y^2$, and the point $P_0(1, 2, 5)$.

a. Find an equation of the tangent plane to S_1 at P_0.

$F(x,y,z) = xyz \Rightarrow \vec{\nabla} F = yz\,\vec{i} + xz\,\vec{j} + xy\,\vec{k}$

$\Rightarrow \vec{n}_1 = \vec{\nabla} F(P_0) = 10\vec{i} + 5\vec{j} + 2\vec{k}$ is normal to S_1 at P_0 (1)

An equation of the tangent plane to S_1 at P_0 is:

$$10\cdot(x-1) + 5\cdot(y-2) + 2\cdot(z-5) = 0$$

b. Find parametric equations of the tangent line to the curve of intersection of S_1 and S_2 at P_0.

$G(x,y,z) = x^2+y^2-z \Rightarrow \vec{\nabla} G = 2x\,\vec{i} + 2y\,\vec{j} - \vec{k}$

$\Rightarrow \vec{n}_2 = \vec{\nabla} G(P_0) = 2\vec{i} + 4\vec{j} - \vec{k}$ is normal to S_2 at P_0 (2)

(1) and (2) $\Rightarrow$ $\vec{v} = \vec{n}_1 \times \vec{n}_2$ is tangent to the curve of intersection of S_1 and S_2 at P_0.

$$\vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 10 & 5 & 2 \\ 2 & 4 & -1 \end{vmatrix} = -13\vec{i} + 14\vec{j} + 30\vec{k}$$

Parametric equations of the tangent line are:

$$\left.\begin{aligned} x &= -13t + 1 \\ y &= 14t + 2 \\ z &= 30t + 5 \end{aligned}\right\}$$

$-\infty < t < \infty$