1. Evaluate the following limits.

a. $\lim_{(x,y)\to(0,0)} \dfrac{xy^5}{x^4+y^6} = 0$ by Sertöz Theorem as $\frac{1}{4}+\frac{5}{6}=\frac{13}{12}>1$

b. $\lim_{(x,y)\to(0,0)} \dfrac{xy^5}{x^4+x^5y+y^6} = \lim_{(x,y)\to(0,0)} \left(\dfrac{xy^5}{x^4+y^6} \cdot \dfrac{x^4+y^6}{x^4+x^5y+y^6} \right)$

$$= \underbrace{\lim_{(x,y)\to(0,0)} \frac{xy^5}{x^4+y^6}}_{=0} \cdot \lim_{(x,y)\to(0,0)} \frac{1}{1+\frac{x^5y}{x^4+y^6}} = 0 \cdot 1 = 0$$

by Part a

$\frac{x^5y}{x^4+y^6} \to 0$ by Sertöz Theorem as $\frac{5}{4}+\frac{1}{6}=\frac{17}{12}>1$

c. $\lim_{(x,y)\to(0,0)} \dfrac{xy^5}{x^4+x^3y+y^6}$

$$\lim_{(x,y)\to(0,0)} \frac{xy^5}{x^4+x^3y+y^6} = \lim_{x\to0} \frac{x\cdot 0^5}{x^4+x^3\cdot 0+0^6} = \lim_{x\to0} 0 = 0$$

along the x-axis

$$\neq$$

$$\lim_{(x,y)\to(0,0)} \frac{xy^5}{x^4+x^3y+y^6} = \lim_{x\to0} \frac{x\cdot(-x)^5}{x^4+x^3\cdot(-x)+(-x)^6} = \lim_{x\to0} -1 = -1$$

along the line $y=-x$

$$\Rightarrow \lim_{(x,y)\to(0,0)} \frac{xy^5}{x^4+x^3y+y^6}$$ does not exist by 2-Path Test