

3. In each of the following, if the given statement is true for all sequences $\{a_n\}_{n=1}^{\infty}$, then mark the \square to the left of TRUE with a \times ; otherwise, mark the \square to the left of FALSE \times and give a counterexample. No explanation is required.

a. If $a_n < a_{n+1}$ for all $n \geq 1$, then $\lim_{n \rightarrow \infty} a_n = \infty$.

TRUE

FALSE, because it does not hold for $a_n =$

$$\frac{1}{n}$$

for $n \geq 1$

b. If $\lim_{n \rightarrow \infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ converges.

TRUE

FALSE, because it does not hold for $a_n =$

$$\frac{1}{n}$$

for $n \geq 1$

c. If $\sum_{n=1}^{\infty} a_n$ converges, then $\{a_n\}_{n=1}^{\infty}$ converges.

TRUE

FALSE, because it does not hold for $a_n =$

$$(-1)^n$$

for $n \geq 1$

d. If $0 < \frac{1}{2^n} < a_n$ for all $n \geq 1$, then $\sum_{n=1}^{\infty} a_n$ diverges.

TRUE

FALSE, because it does not hold for $a_n =$

$$\frac{1}{2^{n-1}}$$

for $n \geq 1$

e. If $0 < a_n < \frac{1}{n}$ for all $n \geq 1$, then $\sum_{n=1}^{\infty} a_n$ converges.

TRUE

FALSE, because it does not hold for $a_n =$

$$\frac{1}{2n}$$

for $n \geq 1$