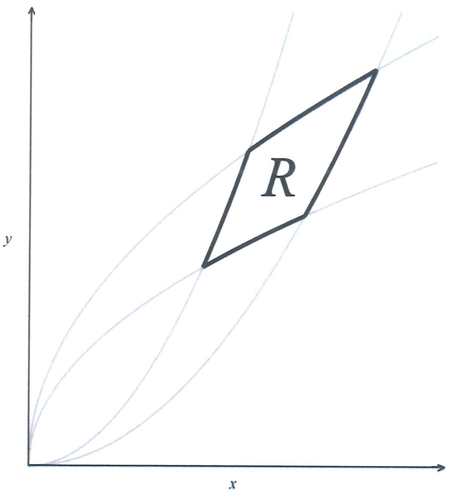
2a. Evaluate the double integral

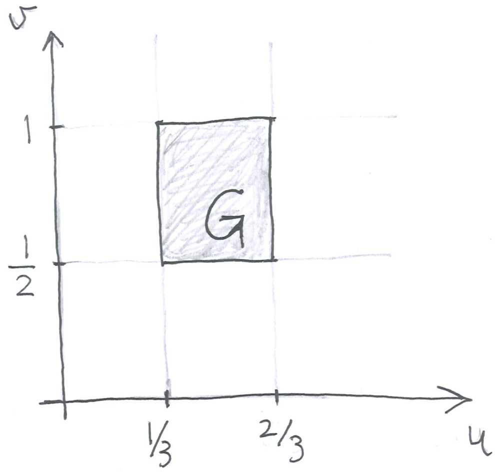
$$\iint_R \cos(\pi x^2/y)\, dA$$

where R is the region bounded by the parabolas $y = 3x^2$, $y = 3x^2/2$, $x = y^2$, $x = 2y^2$ in the plane.



$$\left.\begin{array}{l} u = \dfrac{x^2}{y} \\ v = \dfrac{y^2}{x} \end{array}\right\} \Rightarrow \frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} \frac{2x}{y} & -\frac{x^2}{y^2} \\ -\frac{y^2}{x^2} & \frac{2y}{x} \end{vmatrix} = 4-1=3$$

$$\Rightarrow \frac{\partial(x,y)}{\partial(u,v)} = \frac{1}{\frac{\partial(u,v)}{\partial(x,y)}} = \frac{1}{3}$$



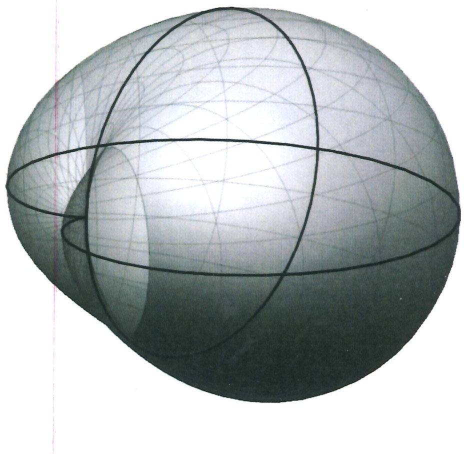
$$\iint_R \cos(\pi x^2/y)\, dA = \iint_G \cos(\pi u) \left|\frac{\partial(x,y)}{\partial(u,v)}\right| dA$$

$$= \int_{1/2}^{1} \underbrace{\int_{1/3}^{2/3} \cos(\pi u) \cdot \frac{1}{3}\, du}_{\substack{\| \\ 0 \text{ by symmetry}}}\, dv = 0$$

2b. A solid D in space satisfies the following conditions:

- The intersection of D with the xy-plane is the region bounded by the cardioid with the equation $r = 1 + \cos\theta$ in polar coordinates.
- The intersection of D with each half-plane $\theta = c$ in spherical coordinates, where c is a constant, is a disk with a diameter lying in the xy-plane.

Express the volume V of the solid D as an iterated integral in spherical coordinates by filling in the rectangles below. No explanation is required.



$$V = \int_{\boxed{0}}^{\boxed{2\pi}} \int_{\boxed{0}}^{\boxed{\pi}} \int_{\boxed{0}}^{\boxed{(1+\cos\theta)\sin\phi}} \boxed{\rho^2 \sin\phi}\, d\rho\, d\phi\, d\theta$$