

3. Each of the following functions has a critical point at  $(0,0)$ . [You do not need to verify this.] Determine whether this critical point is a local maximum, a local minimum, a saddle point or something else.

a.  $f(x,y) = (1-x^2)(1-y^2)$

$$\left. \begin{array}{l} 0 < 1-x^2 \leq 1 \text{ for } |x| < 1 \\ 0 < 1-y^2 \leq 1 \text{ for } |y| < 1 \end{array} \right\} \Rightarrow f(0,0) = 1 \geq (1-x^2)(1-y^2) = f(x,y) \text{ for } x^2+y^2 < 1$$

$\Rightarrow f$  has a local max at  $(0,0)$

b.  $f(x,y) = x^2 - y^4$

$$f(x,0) = x^2 > 0 = f(0,0) \text{ for } x \neq 0$$

$\Rightarrow (0,0)$  is not a local max ①

$$f(0,y) = -y^4 < 0 = f(0,0) \text{ for } y \neq 0$$

$\Rightarrow (0,0)$  is not a local min ②

① and ②  $\Rightarrow (0,0)$  is a saddle point

c.  $f(x,y) = x^2 - xy + y^4$

$$f(y^2, y) = -y^3 + 2y^4 = -y^3 \cdot (1-2y)$$

$$\Rightarrow \left\{ \begin{array}{l} f(y^2, y) > 0 = f(0,0) \text{ for } y < 0 \\ f(y^2, y) < 0 = f(0,0) \text{ for } 0 < y < \frac{1}{2} \end{array} \right. \begin{array}{l} \Rightarrow (0,0) \text{ is not a local max} \\ \Rightarrow (0,0) \text{ is not a local min} \end{array}$$

① ②

① and ②  $\Rightarrow (0,0)$  is a saddle point