2. Suppose that $f(x, y, z)$ is a differentiable function with the gradient

$$\nabla f = (3x^2 - y^2 z)\mathbf{i} - 2xyz\mathbf{j} + (2z - xy^2)\mathbf{k}$$

and consider the point $P_0(1, -1, 2)$.

a. Compute $\nabla f(P_0)$.

$\vec{\nabla} f(P_0) = \vec{i} + 4\vec{j} + 3\vec{k}$

b. Find a unit vector $\mathbf{u}$ for which the directional derivative $D_{\mathbf{u}} f(P_0)$ has its largest possible value.

$\vec{u} =$ "direction of $\vec{\nabla} f(P_0)$" $= \dfrac{\vec{\nabla} f(P_0)}{|\vec{\nabla} f(P_0)|} = \dfrac{\vec{i} + 4\vec{j} + 3\vec{k}}{\sqrt{1^2 + 4^2 + 3^2}} = \dfrac{\vec{i} + 4\vec{j} + 3\vec{k}}{\sqrt{26}}$

c. Find a unit vector $\mathbf{u}$ for which $D_{\mathbf{u}} f(P_0) = 0$.

Take $\vec{u} = \dfrac{4\vec{i} - \vec{j}}{\sqrt{17}}$. Then $D_{\vec{u}} f(P_0) = \vec{\nabla} f(P_0) \cdot \vec{u} = 0$.

d. Find a unit vector $\mathbf{u}$ for which $D_{\mathbf{u}} f(P_0) = 5$.

Take $\vec{u} = \dfrac{4\vec{j} + 3\vec{k}}{5}$. Then $D_{\vec{u}} f(P_0) = \vec{\nabla} f(P_0) \cdot \vec{u} = 5$.

e. Give an example of a function f whose gradient is the one given in this question. No explanation is required.

$f(x, y, z) = \boxed{x^3 - xy^2 z + z^2}$