■ Show all your work and explain your reasoning fully. ■ If you use any tests, clearly state and explicitly check their conditions. ■ Make sure that all your sentences have a subject which is not "it". ■

3. Determine whether each of the following series converges or diverges.

a. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{(\ln n)^{2016}}$

$$c = \lim_{n\to\infty} \frac{\frac{1}{(\ln n)^{2016}}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n}{(\ln n)^{2016}} = \infty$$ by "useful limits"

and $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$ (harmonic series) diverges

$\Rightarrow \displaystyle\sum_{n=2}^{\infty} \frac{1}{(\ln n)^{2016}}$ diverges by the Limit Comparison Test.

b. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{4031}{2016} - n^{1/n}\right)^n$

$$L = \lim_{n\to\infty} \left| \left(\frac{4031}{2016} - n^{1/n}\right)^n \right|^{1/n} = \lim_{n\to\infty} \left(\frac{4031}{2016} - n^{1/n}\right) = \frac{4031}{2016} - 1 = \frac{2015}{2016}$$ by "useful limits"

$L = \frac{2015}{2016} < 1 \Rightarrow \displaystyle\sum_{n=1}^{\infty} \left(\frac{4031}{2016} - n^{1/n}\right)^n$ converges by the n^{th} Root Test.

c. $\displaystyle\sum_{n=1}^{\infty} (2016^{1/n} - 1)$

$$c = \lim_{n\to\infty} \frac{2016^{1/n} - 1}{\frac{1}{n}} = \lim_{x\to\infty} \frac{2016^{1/x} - 1}{\frac{1}{x}} \overset{L'H}{=} \lim_{x\to\infty} \frac{2016^{1/x} \cdot \ln(2016) \cdot \left(-\frac{1}{x^2}\right)}{-\frac{1}{x^2}} = \ln(2016) > 0$$

and $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$ (harmonic series) diverges

$\Rightarrow \displaystyle\sum_{n=1}^{\infty} (2016^{1/n} - 1)$ diverges by the Limit Comparison Test.