

5. Let  $V$  be the volume of the solid cone whose base is the unit disk in the  $xy$ -plane and whose tip is at the point  $(0, 0, 2)$  in the  $xyz$ -space.

a. Only two of ①-③ will be graded. Mark the ones you want to be graded by putting a  $\checkmark$  in the corresponding  $\square$  s.

①  Express  $V$  in terms of iterated integrals in Cartesian coordinates by filling in the rectangles.

$$V = \int_{\square} \int_{\square} \int_{\square} dz dy dx$$

②  Express  $V$  in terms of iterated integrals in cylindrical coordinates by filling in the rectangles.

$$V = \int_0^{2\pi} \int_0^1 \int_0^{2-2r} r dz dr d\theta$$

③  Express  $V$  in terms of iterated integrals in spherical coordinates by filling in the rectangles.

$$V = \int_0^{2\pi} \int_0^{\pi/2} \int_0^{2/(\cos\phi + 2\sin\phi)} \rho^2 \sin\phi d\rho d\phi d\theta$$

b. Compute  $V$  using its expression in terms of iterated integrals in one of the coordinate systems in Part a.

$$\begin{aligned} V &= \int_0^{2\pi} \int_0^1 r \left[ z \right]_{z=0}^{z=2-2r} dr d\theta \\ &= \int_0^{2\pi} \int_0^1 (2r - 2r^2) dr d\theta \\ &= \int_0^{2\pi} \left[ r^2 - \frac{2}{3}r^3 \right]_{r=0}^{r=1} d\theta = \frac{2\pi}{3} \end{aligned}$$

