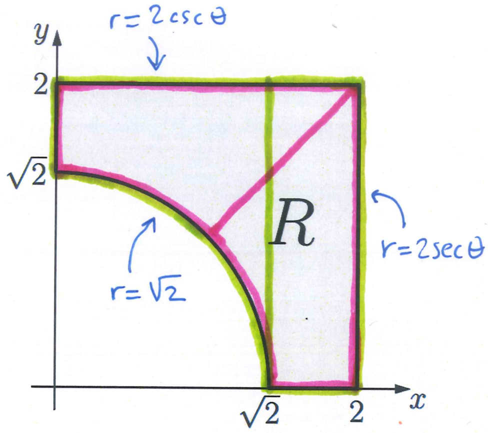
**4.** Consider the double integral

$$I = \iint_R \frac{1}{(x^2+y^2)^2}\, dA$$

where $R$ is the region in the first quadrant lying outside the circle $x^2+y^2=2$, and bounded by the line $x=2$ on the right and the line $y=2$ at the top.




**a.** Express $I$ in terms of iterated double integrals in Cartesian coordinates.

$$I = \int_0^{\sqrt{2}}\int_{\sqrt{2-x^2}}^{2} \frac{1}{(x^2+y^2)^2}\, dy\, dx + \int_{\sqrt{2}}^{2}\int_0^{2} \frac{1}{(x^2+y^2)^2}\, dy\, dx$$

**b.** Express $I$ in terms of iterated double integrals in polar coordinates.

$$I = \int_0^{\pi/4}\int_{\sqrt{2}}^{2\sec\theta} \frac{1}{(r^2)^2}\, r\, dr\, d\theta + \int_{\pi/4}^{\pi/2}\int_{\sqrt{2}}^{2\csc\theta} \frac{1}{(r^2)^2}\, r\, dr\, d\theta$$

**c.** Evaluate $I$.

$$I \overset{\boxed{\text{by symmetry}}}{=} 2\int_0^{\pi/4}\int_{\sqrt{2}}^{2\sec\theta} \frac{1}{r^3}\, dr\, d\theta = 2\int_0^{\pi/4}\left[-\frac{1}{2r^2}\right]_{r=\sqrt{2}}^{r=2\sec\theta} d\theta$$

$$= \int_0^{\pi/4}\left(\frac{1}{2} - \frac{1}{4}\cos^2\theta\right) d\theta = \int_0^{\pi/4}\left(\frac{1}{2} - \frac{1}{4}\cdot\frac{1+\cos 2\theta}{2}\right) d\theta$$

$$= \left[\frac{3}{8}\theta - \frac{1}{16}\sin 2\theta\right]_{\theta=0}^{\theta=\frac{\pi}{4}} = \frac{3\pi}{32} - \frac{1}{16}$$