

3. Suppose that $g(x, y) = f(x^2 - y^2, 2xy)$ where $f(x, y)$ is a differentiable function. Find an equation of the tangent plane to the graph of $z = f(x, y)$ at the point $(3, -4, 7)$ if an equation of the tangent plane to the graph of $z = g(x, y)$ at the point $(-2, 1, 7)$ is $z = 5x - 6y + 23$.

$$\rightarrow g_x(-2, 1) = 5 \quad \text{and} \quad g_y(-2, 1) = -6$$

$$g_x(x, y) = f_x(x^2 - y^2, 2xy) \cdot 2x + f_y(x^2 - y^2, 2xy) \cdot 2y$$

$$g_y(x, y) = f_x(x^2 - y^2, 2xy) \cdot (-2y) + f_y(x^2 - y^2, 2xy) \cdot 2x$$

$$5 = f_x(3, -4) \cdot (-4) + f_y(3, -4) \cdot 2$$

$$-6 = f_x(3, -4) \cdot (-2) + f_y(3, -4) \cdot (-4)$$

$$\rightarrow f_x(3, -4) = -\frac{2}{5} \quad \text{and} \quad f_y(3, -4) = \frac{17}{10}$$

An equation for the tangent plane to the graph of $z = f(x, y)$ at the point $(3, -4, 7)$ is:

$$z = -\frac{2}{5} \cdot (x - 3) + \frac{17}{10} \cdot (y - (-4)) + 7$$