

1. Determine the smallest of the real numbers A, B, C, D, E where :

$$A = \sum_{n=0}^{\infty} \frac{(-1)^n}{2^n} \quad B = \sum_{n=1}^{\infty} \frac{1}{n2^n} \quad C = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n!} \quad D = \sum_{n=1}^{\infty} \frac{n}{3^n} \quad E = \sum_{n=0}^{\infty} \frac{(-1)^n}{3^n(2n+1)}$$

$$A = \sum_{h=0}^{\infty} \left(-\frac{1}{2}\right)^h = \frac{1}{1 - (-\frac{1}{2})} = \frac{2}{3} \quad \leftarrow \text{geometric series with } a=1, r=-\frac{1}{2}$$

$$B = \sum_{h=1}^{\infty} \frac{1}{n2^n} > \frac{1}{2} + \frac{1}{2 \cdot 2^2} + \frac{1}{3 \cdot 2^3} = \frac{2}{3}$$

$$D = \sum_{h=1}^{\infty} \frac{n}{3^n} > \frac{1}{3} + \frac{2}{3^2} + \frac{3}{3^3} = \frac{2}{3}$$

$$E = \sum_{h=0}^{\infty} \frac{(-1)^h}{3^h(2h+1)} > 1 - \frac{1}{3 \cdot 3} = \frac{8}{9} > \frac{2}{3}$$

$$C = \sum_{h=1}^{\infty} \frac{(-1)^{h+1}}{h!} < 1 - \frac{1}{2!} + \frac{1}{3!} = \frac{2}{3}$$

C is the smallest.