4. In Genetics, *Fisher's Equation*,

$$\frac{\partial p}{\partial t} = p\,(1-p) + \frac{\partial^2 p}{\partial x^2}$$

describes the spread of an advantageous allele in a population with uniform density along a 1-dimensional habitat, like a shoreline, as a result of both reproduction and dispersion of the offspring. Here $p(x,t)$ is the frequency of the allele as a function of the position x and the time t.

Find all possible values of the pair of constants (a,b) for which the function

$\times (1+e^{ax+bt})^4 \cdot e^{-(ax+bt)}$

$$p(x,t) = \frac{1}{(1+e^{ax+bt})^2}$$

satisfies the Fisher's Equation.

Compute carefully!

$$p_t = -2(1+e^{ax+bt})^{-3} \cdot e^{ax+bt} \cdot b$$

$$p_x = -2(1+e^{ax+bt})^{-3} \cdot e^{ax+bt} \cdot a$$

$$p_{xx} = 6(1+e^{ax+bt})^{-4} \cdot (e^{ax+bt} \cdot a)^2 - 2(1+e^{ax+bt})^{-3} \cdot e^{ax+bt} \cdot a^2$$

$$p \cdot (1-p) = (1+e^{ax+bt})^{-4} \cdot \left(2e^{ax+bt} + (e^{ax+bt})^2\right)$$

$$-2b(1+e^{ax+bt}) = 2 + e^{ax+bt} + 6a^2 e^{ax+bt} - 2a^2(1+e^{ax+bt})$$

$$\Downarrow$$

$$2a^2 - 2b - 2 = (4a^2+2b+1)\,e^{ax+bt} \quad \text{for all } (x,t)$$

$$\begin{cases} 2a^2-2b-2=0 \\ 4a^2+2b+1=0 \end{cases} \Longrightarrow 6a^2 = 1 \Longrightarrow a = \frac{1}{\sqrt{6}} \text{ or } a = -\frac{1}{\sqrt{6}}$$

$$b = -\frac{5}{6}$$

$$\Longrightarrow (a,b) = \left(\frac{1}{\sqrt{6}}, -\frac{5}{6}\right), \left(-\frac{1}{\sqrt{6}}, -\frac{5}{6}\right)$$