3a. Evaluate the following limits. [Show all your work.]

❶ $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^3y^2}{x^6+y^6}$

$\lim\limits_{(x,y)\to(0,0)} \dfrac{x^3y^2}{x^6+y^6} = \lim\limits_{x\to 0} \dfrac{x^3\cdot x^2}{x^6+x^6} = \dfrac{1}{2}\lim\limits_{x\to 0}\dfrac{1}{x}$ does not exist

along the line $y=x$

$\Rightarrow \lim\limits_{(x,y)\to(0,0)} \dfrac{x^3y^2}{x^6+y^6}$ does not exist by the 1-Path Test.

❷ $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^3y^2}{x^6 \boxed{+} x^2y^2+y^6}$

☜ Your choice: Fill in the ☐ with "+" or "−", then evaluate!

$x^6\geq 0,\ x^2y^2\geq 0,\ y^6\geq 0 \Rightarrow 0\leq x^2y^2\leq x^6+x^2y^2+y^6$

$\Rightarrow 0\leq \dfrac{x^2y^2}{x^6+x^2y^2+y^6}\leq 1 \Rightarrow 0\leq \left|\dfrac{x^3y^2}{x^6+x^2y^2+y^6}\right|\leq |x|$ for $(x,y)\neq(0,0)$

$\to 0$ as $(x,y)\to(0,0)$

$\Rightarrow \lim\limits_{(x,y)\to(0,0)} \dfrac{x^3y^2}{x^6+x^2y^2+y^6}=0$ by the Sandwich Theorem.

3b. Evaluate the following statements. [Just check and fill, no further explanation is required.]

❸ If $f(x,y)$ approaches 0 as (x,y) approaches $(0,0)$ along any line through $(0,0)$, then $\lim\limits_{(x,y)\to(0,0)} f(x,y)=0$.

☐ TRUE and can be proven using the [ ]

☑ FALSE because does not hold for $f(x,y)=$ $\dfrac{xy^2}{x^2+y^4}$

❹ If, for $(x,y)\neq(0,0)$, the line $y=x$ and every circle tangent to it at $(0,0)$ are level curves of $f(x,y)$ belonging to different values, then $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ does not exist.

☑ TRUE and can be proven using the 2-Path Test

☐ FALSE because does not hold for $f(x,y)=$ [ ]