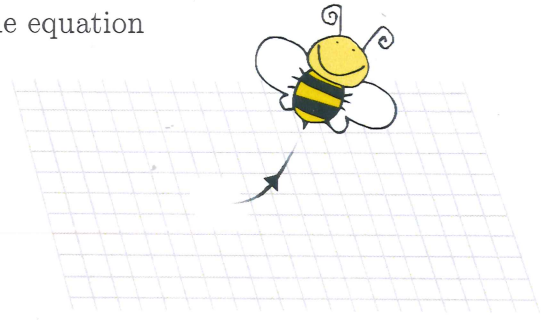**2.** In the xyz-space where a flyscreen lies along the plane with the equation

$$2x + y - 2z = 1 \,,$$

the trajectory of a bee as a function of time t is given by

$$\mathbf{r} = t\,\mathbf{i} + t^2\,\mathbf{j} + t^3\,\mathbf{k}$$

for $-\infty < t < \infty$.

a. Find the velocity $\mathbf{v}$ of the bee as a function of time.

$$\vec{v} = \vec{i} + 2t\,\vec{j} + 3t^2\,\vec{k}$$

b. Give an example of a nonzero vector $\mathbf{n}$ perpendicular to the screen.

$$\vec{n} = 2\vec{i} + \vec{j} - 2\vec{k}$$

c. Find all times t when the bee is flying parallel to the screen.

$$\Longrightarrow \quad \vec{v} \cdot \vec{n} = 0 \Rightarrow 2 + 2t - 6t^2 = 0 \Rightarrow t = \frac{1 \pm \sqrt{13}}{6}$$

d. Find all times t when the bee is flying perpendicular to the screen.

$$\Longrightarrow \quad \vec{v} \,/\!/\, \vec{n} \Longrightarrow \frac{1}{2} = \frac{2t}{1} = \frac{3t^2}{-2} \Longrightarrow t^2 = -\frac{1}{3} \Longrightarrow \text{No such } t$$

e. There are holes in the screen through which the bee passes. Find the coordinates of all of these holes.

$$\left.\begin{array}{l} x = t \\ y = t^2 \\ z = t^3 \end{array}\right\} \text{ and } 2x + y - 2z = 1 \Longrightarrow 2t + t^2 - 2t^3 = 1$$

$$\Longrightarrow 2t^3 - t^2 - 2t + 1 = 0 \Longrightarrow (2t-1)(t^2-1) = 0$$

$$\Longrightarrow \quad t = \frac{1}{2} \quad \text{or} \quad t = 1 \quad \text{or} \quad t = -1$$

$$\Longrightarrow (x, y, z) = \left(\frac{1}{2}, \frac{1}{4}, \frac{1}{8}\right), \quad (1, 1, 1), \quad (-1, 1, -1)$$