3. Determine whether each of the following series converges or diverges.

Explain your reasoning in full by clearly stating the name and the conditions of the test you are using, and explicitly checking the validity of these conditions for the given series.

a. $\sum_{n=1}^{\infty} (5^n - 3^n)^{-1}$

$$L = \lim_{n\to\infty} \frac{\frac{1}{5^n - 3^n}}{\frac{1}{5^n}} = \lim_{n\to\infty} \frac{1}{1 - \underbrace{(3/5)^n}_{\to 0}} = 1$$

$0 < 1 < \infty$ and $\sum_{n=1}^{\infty} \frac{1}{5^n}$ converges (because it is a geometric series with $|r| = \left|\frac{1}{5}\right| = \frac{1}{5} < 1$)

$\Rightarrow \sum_{n=1}^{\infty} \frac{1}{5^n - 3^n}$ converges by the Limit Comparison Test.

b. $\sum_{n=2}^{\infty} \sin^5(\pi/\sqrt[3]{n})$

$$L = \lim_{n\to\infty} \frac{\sin^5\left(\frac{\pi}{\sqrt[3]{n}}\right)}{\frac{1}{n^{5/3}}} = \pi^5 \Bigg(\underbrace{\lim_{n\to\infty} \frac{\sin\left(\frac{\pi}{n^{1/3}}\right)}{\frac{\pi}{n^{1/3}}}}_{1}\Bigg)^5 = \pi^5$$

$0 < \pi^5 < \infty$ and $\sum_{n=1}^{\infty} \frac{1}{n^{5/3}}$ converges (because it is a p-series with $p = \frac{5}{3} > 1$)

$\Rightarrow \sum_{n=2}^{\infty} \sin^5\left(\frac{\pi}{\sqrt[3]{n}}\right)$ converges by the Limit Comparison Test.

c. $\sum_{n=1}^{\infty} \cos^5(\pi/\sqrt[3]{n})$

$$\lim_{n\to\infty} \cos^5\left(\frac{\pi}{\sqrt[3]{n}}\right) = \cos^5(0) = 1 \neq 0$$

$\Rightarrow \sum_{n=1}^{\infty} \cos^5\left(\frac{\pi}{\sqrt[3]{n}}\right)$ diverges by the n^{th} Term Test.