2. In each of the following, if the reasoning in the given sentence is correct, then ✔ the corresponding ☐; otherwise, leave it blank. No explanation is required.

For each of the parts (**a-e**), to get full points you must check *exactly* the squares corresponding to the correct reasonings. Note that in each sentence all the statements to the right of "because" are true. You must decide whether they lead to the statement to the left of "because", possibly using a test you have seen in this course.

a. ☐ $\sum_{n=1}^{\infty} \frac{1}{n}$ converges because $\lim_{n\to\infty} \frac{1}{n} = 0$.

☐ $\sum_{n=1}^{\infty} \frac{1}{n}$ converges because $0 < \frac{1}{n+1} < \frac{1}{n}$ for all $n \geq 1$.

☑ $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges because $1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} > \ln(n+1)$ for all $n \geq 1$.

☑ $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges because $1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{2^n} > 1 + \frac{n}{2}$ for all $n \geq 2$.

b. ☐ $\sum_{n=0}^{\infty} (-1)^n$ converges because $-1 \leq (-1)^n \leq 1$ for all $n \geq 0$.

☑ $\sum_{n=0}^{\infty} (-1)^n$ diverges because the sequence $1, -1, 1, -1, 1, \ldots$ diverges.

☑ $\sum_{n=0}^{\infty} (-1)^n$ diverges because the sequence $1, 0, 1, 0, 1, \ldots$ diverges.

☐ $\sum_{n=0}^{\infty} (-1)^n$ diverges because $-1 \leq (-1)^n \leq 1$ for all $n \geq 0$.

c. ☐ $\sum_{n=1}^{\infty} \frac{1}{n^{1+1/n}}$ converges because $\lim_{n\to\infty} \frac{1}{n^{1+1/n}} = 0$.

☐ $\sum_{n=1}^{\infty} \frac{1}{n^{1+1/n}}$ converges because $0 < (1/(n+1)^{1+1/(n+1)})/(1/n^{1+1/n}) < 1$ for all $n \geq 1$.

☐ $\sum_{n=1}^{\infty} \frac{1}{n^{1+1/n}}$ converges because $0 < \frac{1}{n^{1+1/n}} < \frac{1}{n}$ for all $n \geq 2$ and $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges.

☑ $\sum_{n=1}^{\infty} \frac{1}{n^{1+1/n}}$ diverges because $\lim_{n\to\infty} ((1/n^{1+1/n})/(1/n)) = 1$ and $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges.

d. ☑ $\sum_{n=2}^{\infty} \left(1 - \frac{1}{n}\right)^{n^2}$ converges because $\lim_{n\to\infty} \left(\left(1 - \frac{1}{n}\right)^{n^2}\right)^{1/n} = e^{-1} < 1$.

☐ $\sum_{n=2}^{\infty} \left(1 - \frac{1}{n}\right)^{n^2}$ converges because $\lim_{n\to\infty} \left(1 - \frac{1}{n}\right)^{n^2} = 0$.

☑ $\sum_{n=2}^{\infty} \left(1 - \frac{1}{n}\right)^{n^2}$ converges because $0 < \left(1 - \frac{1}{n}\right)^{n^2} < \frac{1}{2^n}$ for all $n \geq 2$ and $\sum_{n=0}^{\infty} \frac{1}{2^n}$ converges.

☐ $\sum_{n=2}^{\infty} \left(1 - \frac{1}{n}\right)^{n^2}$ diverges because $\frac{1}{3^n} < \left(1 - \frac{1}{n}\right)^{n^2}$ for all $n \geq 6$ and $\sum_{n=0}^{\infty} \frac{1}{3^n}$ converges.

e. ☑ $\sum_{n=0}^{\infty} \frac{1}{2^n}$ converges because $\lim_{n\to\infty} \left(\frac{1}{2^n}\right)^{1/n} = \frac{1}{2} < 1$.

☑ $\sum_{n=0}^{\infty} \frac{1}{2^n}$ converges because $\lim_{n\to\infty} ((1/2^{n+1})/(1/2^n)) = \frac{1}{2} < 1$.

☑ $\sum_{n=0}^{\infty} \frac{1}{2^n}$ converges because the sequence $1, \frac{3}{2}, \frac{7}{4}, \frac{15}{8}, \frac{31}{16}, \ldots$ converges.

☐ $\sum_{n=0}^{\infty} \frac{1}{2^n}$ diverges because $\frac{1}{2^n} \neq 0$ for infinitely many $n \geq 0$.