

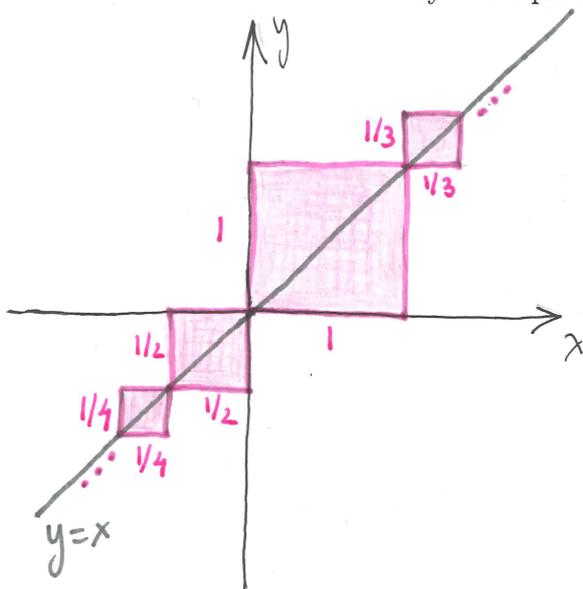
We have a $1/n \times 1/n$ square for each positive integer n . For each of (a-c), indicate by ing the to the left of YES or No whether it is possible or not to place these squares in the xy -plane in such a way that they completely cover the given set. If YES, describe how this can be done (you might also want to draw a picture) and then fully justify your claim. If No, explain why this cannot be done.

a. The entire xy -plane : YES NO

The total area of the squares is finite as $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$,

but the area of the entire plane is infinite.

b. The line defined by the equation $y = x$: YES NO

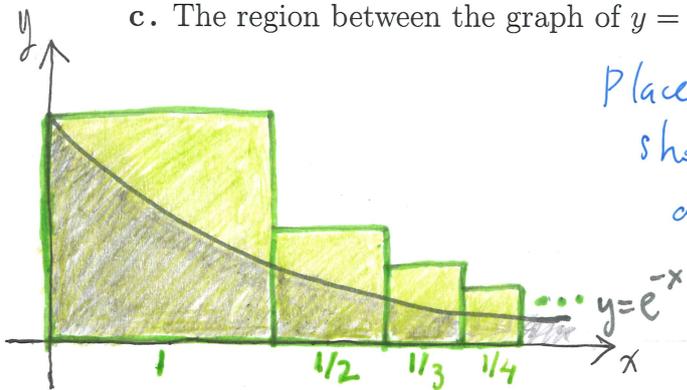


Place the odd-numbered squares in the 1st quadrant and the even-numbered squares in the 3rd quadrant along the line as shown. Then they cover the entire line as

$$\sum_{n=1}^{\infty} \frac{\sqrt{2}}{2n} = \frac{1}{\sqrt{2}} \sum_{n=1}^{\infty} \frac{1}{n} = \infty \text{ and}$$

$$\sum_{k=1}^{\infty} \frac{\sqrt{2}}{2k-1} > \sum_{k=1}^{\infty} \frac{\sqrt{2}}{2k} = \infty$$

c. The region between the graph of $y = e^{-x}$ and the x -axis for $x \geq 0$: YES NO



Place the squares along the positive x -axis as shown. Then they extend along the entire axis as $\sum_{n=1}^{\infty} \frac{1}{n} = \infty$. On the other hand, applying the Integral Test inequality to

the function $f(x) = \frac{1}{x}$ (which is positive, continuous and decreasing on $[1, \infty)$) we obtain $1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} > \ln(n+1)$ for $n > 0$. Hence for $\sum_{k=1}^n \frac{1}{k} \leq x \leq \sum_{k=1}^{n+1} \frac{1}{k}$, $e^{-x} \leq e^{-(1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n})} \leq \frac{1}{n+1}$ for $n \geq 0$. In other words, the top side of the $(n+1)$ st square lies above the graph, and the squares cover the region.