

5. Let V be the volume of the ball $B = \{(x, y, z) : x^2 + y^2 + z^2 \leq 2z\}$.


a. Only two of ①-③ will be graded. Mark the ones you want to be graded by putting a \checkmark in the corresponding \square s.

① Express V in terms of iterated integrals in Cartesian coordinates by filling in the rectangles.

$$V = \int_{\square} \int_{\square} \int_{\square} dz dy dx$$

② Express V in terms of iterated integrals in cylindrical coordinates by filling in the rectangles.

$$V = \int_0^0 \int_0^{2\pi} \int_0^{\sqrt{2z-z^2}} r dr dz d\theta$$

Note the order 

③ Express V in terms of iterated integrals in spherical coordinates by filling in the rectangles.

$$V = \int_0^0 \int_0^{\pi/2} \int_0^{2\cos\phi} \rho^2 \sin\phi d\rho d\phi d\theta$$

b. Compute V using its expression in terms of iterated integrals in one of the coordinate systems in part (a).

$$\begin{aligned} V &= \int_0^{2\pi} \int_0^2 \int_0^{\sqrt{2z-z^2}} r dr dz d\theta = \int_0^{2\pi} \int_0^2 \left[\frac{1}{2} r^2 \right]_{r=0}^{r=\sqrt{2z-z^2}} dz d\theta \\ &= \int_0^{2\pi} \int_0^2 (z - \frac{1}{2} z^2) dz d\theta = \int_0^{2\pi} \left[\frac{1}{2} z^2 - \frac{1}{6} z^3 \right]_{z=0}^{z=2} d\theta \\ &= 2\pi \cdot \frac{2}{3} = \frac{4\pi}{3} \end{aligned}$$

