3. Find the absolute maximum and minimum values of the function $f(x,y) = 2(x^2+y^2-1)^2 + x^2 - y^2$ on the unit disk $D = \{(x,y) : x^2+y^2 \le 1\}$.

Interior of D:

$$\left.\begin{aligned} f_x &= 2\cdot 2(x^2+y^2-1)\cdot 2x + 2x = 0 \\ f_y &= 2\cdot 2(x^2+y^2-1)\cdot 2y - 2y = 0 \end{aligned}\right\}$$

$(x=0$ or $4x^2+4y^2=3)$ and $(y=0$ or $4x^2+4y^2=5)$

No solution

Critical points: $(x,y) = (0,0), \left(\pm\frac{\sqrt{3}}{2}, 0\right), \cancel{\left(0, \pm\frac{\sqrt{5}}{2}\right)}$ not in D

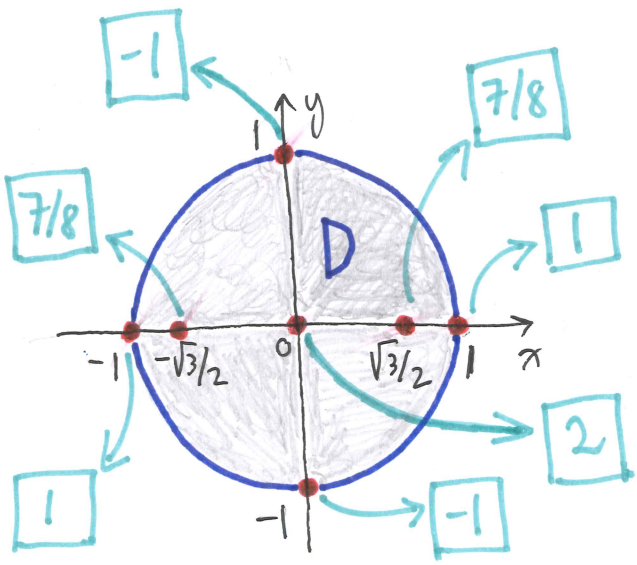
Boundary of D: $x^2+y^2=1 \Rightarrow y^2 = 1-x^2$

Consider $f(x, \pm\sqrt{1-x^2}) = 2x^2-1$ for $-1 \le x \le 1$.

$\frac{d}{dx} f(x, \pm\sqrt{1-x^2}) = 4x = 0$

Critical points: $x=0 \Rightarrow (x,y) = (0, \pm 1)$

Endpoints: $x = \pm 1 \Rightarrow (x,y) = (\pm 1, 0)$



Absolute max is 2,

absolute min is -1.