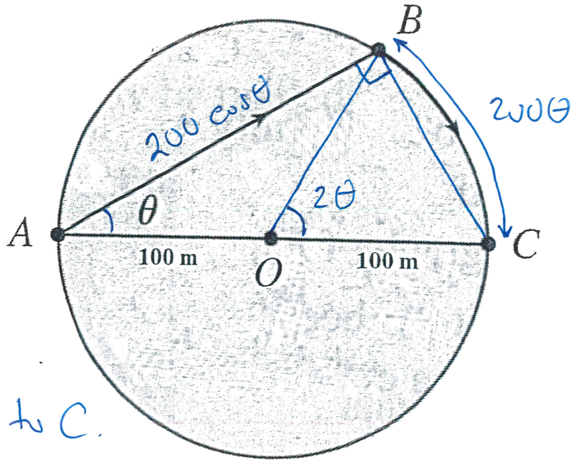
1. You are standing at a point A on the shore of a circular lake with radius 100 m, and you are planning to go to the point C, where $[AC]$ is a diameter of the circle. You will swim from A to a point B on the shore along a straight line and then either walk or run from B to C along the shore.

- You can swim with a speed of 1 m/s.
- You can walk with a speed of $\sqrt{2}$ m/s.
- You can run with a speed of 2 m/s.

Determine the angle $\theta = \widehat{CAB}$ that will take you from A to C in the shortest possible time

a. if you walk from B to C, and

b. if you run from B to C.



Let T be the time it takes to go from A to C.
Let v be the velocity on land. Then:

$$T = 200\cos\theta + \frac{200\,\theta}{v} = 200\cdot\left(\cos\theta + \frac{\theta}{v}\right) \quad \text{for } 0 \le \theta \le \frac{\pi}{2}$$

Hence:

$$\frac{dT}{d\theta} = 200\cdot\left(-\sin\theta + \frac{1}{v}\right) = 0 \Rightarrow \sin\theta = \frac{1}{v}$$

(a) If you walk, then $v = \sqrt{2}$ m/s.

Critical point: $\sin\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{\pi}{4} \Rightarrow T = 200\cdot\left(\frac{1}{\sqrt{2}} + \frac{\pi}{4\sqrt{2}}\right)$

Endpoints: $\theta = 0 \Rightarrow T = 200$

$\theta = \frac{\pi}{2} \Rightarrow T = 200\cdot\frac{\pi}{2\sqrt{2}}$

Shortest time occurs for $\theta = 0$ as $\pi > 3 > 2\sqrt{2}$ and $4 + \pi > 7 > 4\sqrt{2}$.

(b) If you run, then $v = 2$ m/s.

Critical point: $\sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6} \Rightarrow T = 200\cdot\left(\frac{\sqrt{3}}{2} + \frac{\pi}{12}\right)$

Endpoints: $\theta = 0 \Rightarrow T = 200$

$\theta = \frac{\pi}{2} \Rightarrow T = 200\cdot\frac{\pi}{4}$

Shortest time occurs for $\theta = \frac{\pi}{2}$ as $4 > \pi$ and $6\sqrt{3} > 8 > 2\pi$.